

CO2013: Complex Analysis, Quiz-1, Fall 2016

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Notice:

- a) *Term grading policy: Quiz-1* $\times 10\%$.
- b) **Total 100 points** in this exam.
- c) **Exam Time: 9:00AM–9:55AM**, Oct. 28, 2016.

1. **(10 pts)** Verify the identity

$$\sin z_2 - \sin z_1 = 2 \cos \left(\frac{z_2 + z_1}{2} \right) \sin \left(\frac{z_2 - z_1}{2} \right),$$

and find the relationship between z_1 and z_2 if $\sin z_1 = \sin z_2$.

2. **(20 pts)** Solve the following equations:

(a) $\text{Log}(z^2 - 1) = \frac{i\pi}{2}$. (b) $e^{2z} + e^z + 1 = 0$.

3. **(20 pts)** Find the following values:

(a) $(1 + i)^{1-i}$. (b) the principal value of $(1 + i)^{1+i}$.

4. **(15 pts)** Prove that $f(z) = \bar{z}$ is continuous on the whole plane but is nowhere differentiable.

5. **(10 pts)** Let $z = x + iy$, where $x, y \in \mathbb{R}$, show that

$$f(z) = e^{x^2 - y^2} [\cos(2xy) + i \sin(2xy)]$$

is entire, and find its derivative.

6. **(10 pts)** Find a harmonic conjugate of $u(x, y)$ with $u(x, y) = \sin x \cosh y$.

7. **(15 pts)** Use the De Moivre's formula to establish the identity (you NEED TO give complete derivation to the result!),

$$1 + \cos \theta + \cos 2\theta + \cdots + \cos n\theta = \frac{1}{2} + \frac{\sin[(n + \frac{1}{2})\theta]}{2 \sin(\theta/2)}.$$