CO2013: Complex Analysis, Quiz-1, Fall 2018

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Notice:
a) Term grading policy: Quiz-1×10%.
b) Total 100 points in this exam.
c) Exam Time: 1:00PM−2:50PM, Oct. 19, 2018.

1. (10 pts) Find z such that $e^z = (1+i)/\sqrt{2}$.

2. (5+10 pts) Solve the following equations:
   (a) $e^z = 2i$. (b) $e^{2z} + e^z + 1 = 0$.

3. (10+10 pts)
   (a) Show that $\sin^{-1} z = -i \log[iz + (1-z^2)^{1/2}]$.
   (b) Find the solutions of the equation $\sin z = 2$.

4. (15 pts) Show that $f(z) = |z|^2$ is differentiable at $z = 0$ but is not differentiable at any other point.

5. (20 pts) If u and v are expressed in terms of polar coordinates (r, θ), show that the Cauchy-Riemann equations can be written in the form

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}, \qquad \frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}.$$

6. (10 pts) Find a harmonic conjugate of $u = \ln|z|$ for $\mathrm{Re}\{z\} > 0$.

7. (5+5 pts)
   (a) Find all the values of $\left(\frac{2i}{1+i}\right)^{1/6}$.
   (b) Solve the equation $z^2 - 2z + i = 0$.