

Notice:

- a) Term grading policy: Quiz-3  $\times$  15%.
- b) Total 100 points in this exam.
- c) Exam Time: 1:00PM–2:50PM, Dec. 28, 2018.

1. Let  $f(z) = 1/(\sqrt{2} \cos z - 1 + z - \frac{\pi}{4})^2$ .

(a) (10 pts) Find the order of the pole of  $f(z)$  at  $z = \frac{\pi}{4}$ .

(b) (10 pts) Evaluate  $\oint_C \frac{f'(z)}{f(z)} dz$ , where  $C : |z - \frac{\pi}{4}| = 1$ .

2. (20 pts) Evaluate each of the following contour integrals:

(a)  $\oint_{|z|=1} \cot z dz$ ,

(b)  $\oint_{|z|=1} \frac{\csc z}{z^2} dz$ ,

(c)  $\oint_{|z|=4} \frac{z-1}{\sin z} dz$ ,

(d)  $\oint_{|z|=2} \left( \frac{z-1}{z+1} \right)^3 dz$ .

3. (15 pts) Find the results for  $\omega$  in the upper or lower half-plane,

$$\text{P.V.} \int_{-\infty}^{\infty} \frac{\cos x}{x - \omega} dx.$$

4. (45 pts) Evaluate each of the following integrals:

(a)  $\int_0^\pi \frac{d\theta}{(3 + 2 \cos \theta)^2}$ , (b)  $\int_0^\infty \frac{x^2 + 1}{x^4 + 1} dx$ , (c)  $\text{P.V.} \int_{-\infty}^{\infty} \frac{x}{(x^2 + 4x + 13)^2} dx$ .