CO2013: Complex Analysis, Exam-3, Fall 2019

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Notice:
a) Term grading policy: Exam-3×25%.
b) Total 100 points in this exam.
c) Exam Time: 1:00PM−2:50PM, Dec. 13, 2019.

1. (15 pts) Find the radius of convergence in the following series:

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n2^n}(z-1-i)^n$, (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(z-4-3i)^n}{5^{2n}}$,

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{(2n)!}{(n+2)(n!)^2}(z-i)^{2n}$.

2. (10 pts) Determine whether the given sequence converges or diverges:

(a) $\displaystyle\left\{\frac{n(1+i^n)}{n+1}\right\}$, (b) $\left\{e^{1/n} + 2(\tan^{-1} n)i\right\}$.

3. (10 pts) Let $f(z) = \displaystyle\sum_{n=0}^{\infty}(n^3/3^n)z^n$. Compute the following

(a) $\displaystyle\oint_{|z|=1} \frac{f(z)}{z^4}dz$, (b) $\displaystyle\oint_{|z|=1} \frac{f(z)\sin z}{z^3}dz$.

4. (10 pts) Suppose the function $f(z) = \dfrac{3-i}{1-i+z}$ is expanded in a Taylor series with center $4-2i$. What is the radius of convergence?

5. (10 pts) Find the first three nonzero terms in Maclaurin expansion of

$$f(z) = \int_0^z e^{\tau^3} d\tau.$$

Notice: For problems 6-8, you need to give a closed form or explicitly write out at least the first four terms of the power series as your answer to each question. For example,

$$e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!} = 1 + z + \frac{z^2}{2} + \frac{z^3}{6} + \cdots.$$

6. (10 pts) Expand $f(z) = \dfrac{z}{(1-z)^3}$ in a Maclaurin series and give the radius of convergence.

7. (15 pts) Expand $f(z) = \dfrac{1+z}{1-z}$ in a Taylor series centered at $z_0 = i$ and give the radius of convergence.

8. (20 pts) Expand the following functions in Laurent series:
   (a) $f(z) = \dfrac{1}{(z-1)^2(z-3)}$ for $0 < |z-1| < 2$, and
   (b) $f(z) = \dfrac{1}{z(z-1)}$ for $1 < |z-2| < 2$.