

Notice:

- a) Term grading policy: Final \times **35%**.
- b) Total 125 points in this exam.
- c) Exam Time: 1:00PM–3:20PM, 13 Jan., 2022.

1. (25 pts) Evaluate the integral $\int_0^{2\pi} \frac{\cos \theta}{2 + \sin \theta} d\theta$.

2. (25 pts) Evaluate the integral P.V. $\int_{-\infty}^{\infty} \frac{x \sin x}{(x^2 + 1)(x^2 - 3x + 2)} dx$.

3. (25 pts) Evaluate the integral P.V. $\int_0^{\infty} \frac{dx}{\sqrt{x}(x^2 - 3x - 4)}$.

4. (25 pts) Find the piecewise smooth function $f(t)$ for $t > 0$ with Laplace transform of $F(s) = \mathcal{L}\{f(t)\} = \frac{1}{s^4 - 2s^3 + 2s^2 - 2s + 1}$ using the residue integration method.

5. (10+15 pts) (a) Find the Fourier transform of $f(t) = u(t)te^{-2t}$, where $u(t)$ is the step function, and (b) verify the inversion formula obtained in (a) using the residue integration method. (Hint: you may need the L'Hopital's rule.)