

Notice:

- a) Term grading policy: Exam-1  $\times$  15%.
  - b) Total 100 points in this exam.
  - c) Exam Time: 1:00PM–2:50PM, Oct. 21, 2021.
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1. (20 pts) Find all the vales of (a)  $(-8 - 8\sqrt{3}i)^{1/4}$  and (b)  $(2 + 2i)^{2/3}$ .
  2. (10 pts) Use De Moivre's formula to find the identity of  $\cos 4\theta$  in terms of  $\cos \theta$ .
  3. (10 pts) Find all the solutions of  $z^5 + 2z^4 + 4z^3 + 8z^2 + 16z + 32 = 0$  in the form of  $x + iy$ .
  4. (10 pts) Explain the analyticity of  $f(z) = e^{i\bar{z}}$ .
  5. (10 pts) Show that if  $v$  is a harmonic conjugate for  $u$ , then  $-u + C$  is a harmonic conjugate for  $v$ , where  $C$  is a constant.
  6. (20 pts) (a) Verify that  $u = \text{Im}\{e^{z^2}\}$  is harmonic and (b) find a harmonic conjugate for  $u$ .
  7. (20 pts) (a) Find the most general harmonic polynomial of the form  $u(x, y) = \alpha x^3 + \beta x^2y + \gamma xy^2 + \delta y^3$ . (Remove different notations of coefficients  $(\alpha, \beta, \gamma, \delta)$  as many as possible if they are related.) (b) Find a harmonic conjugate for  $u(x, y)$ .