

## CO2013: Complex Analysis, Final, Fall 2016

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Notice:

a) *Term grading policy: Final*  $\times 40\%$ .

b) **Total 165 points (Take as many as you want!) in this exam.**

c) **Exam Time: 10:00AM–12:30PM, Jan. 9, 2017.**

1. **(15 pts)** Determine where the following complex mapping  $w = f(z)$  is conformal.

$$(a) f(z) = z^3 + 3z + 1 \quad (b) f(z) = z - e^{-z} + i \quad (c) f(z) = \tan z$$

2. **(10 pts)** Construct a Mobius transformation that maps the points  $-i, 1$ , and  $\infty$  on the line  $y = x - 1$  onto the points  $1, i$  and  $-1$  onto the unit circle  $|w| = 1$ .

3. **(10 pts)** Find the image of the exterior of the circle  $C : |z - 2| = 2$  under the Mobius transformation

$$w = f(z) = \frac{z}{z - 4}.$$

4. **(10 pts)** What is the order of the pole of

$$f(z) = \frac{1}{(\sin z - z + z^2)^2}$$

at  $z = 0$ ?

5. **(10 pts)** Let  $f(z) = \frac{z^2(z+i)^4(z-3)^6 e^{z^2}}{3(z-1)^3(2z-7)^5}$ , evaluate  $\oint_{D_2^+(0)} \frac{f'(z)}{f(z)} dz$ .

6. **(20 pts)** Evaluate each of the following contour integrals with positive orientation:

$$(a) \oint_{|z|=1} e^{1/z} \tan\left(\frac{1}{2z}\right) dz, \quad (b) \oint_{|z|=1} \frac{e^z}{z^3 + 5z^2} dz.$$

7. **(20 pts)** Evaluate

$$\int_0^\infty \frac{\log x}{(x^2 + 1)^2} dx.$$

8. **(30 pts)** Show that  $-1 < \alpha < 1$ ,

$$\text{P.V.} \int_0^\infty \frac{x^\alpha}{x^2 - 1} dx = \frac{\pi[1 - \cos(\pi\alpha)]}{2 \sin(\pi\alpha)}.$$

**Notice: You have to write down complete analysis to all parts of the integral (tell the reason even if the value of the part is zero) in Problem 8.**

9. **(40 pts)** Evaluate each of the following integrals:

$$(a) \int_0^{2\pi} \frac{\cos 2\theta}{13 - 12 \cos \theta} d\theta, \quad (b) \text{P.V.} \int_{-\infty}^\infty \frac{x \cos x}{x^2 - 3x + 2} dx.$$