CO2013: Complex Analysis, Midterm, Fall 2016

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Notice:
a) *Term grading policy:* **Midterm**$\times 30\%$.
b) *Total* **100 points (2 pages, see the next page for Problems 6-8!)** *in this exam.*
c) *Exam Time*: **10:00AM−11:50AM**, Nov. 28, 2016.

1. (**10 pts**) By using the formula $e^{iz} = \cos z + i \sin z$, solve the equation
$$\sin z = 2$$
for z in terms of natural logarithm.

2. (**12 pts**) (a) Verify that $u(x, y) = x^3 - 3xy^2 - 5y$ is harmonic in the entire complex plane. (b) Find a harmonic conjugate of $u(x, y)$.

3. (**12 pts**) If $f(z) = \sum_{n=0}^{\infty} \frac{n^4}{4^n} z^n$ and C is the circle $|z| = 1$ traversed once in the positive sense, compute
$$\oint_C \frac{(z+1)f(z)}{z^5} dz.$$

4. (**15 pts**) Explain and determine if the given series converges or diverges in the following:

(a) $\sum_{n=2}^{\infty} \frac{i^n}{(1+i)^{n-1}}$, (b) $\sum_{n=1}^{\infty} \frac{(1+i)^n}{(-1)^n n^3}$,

(c) $\sum_{n=1}^{\infty} \frac{i^n}{n^2}$, (d) $\sum_{n=0}^{\infty} \frac{i^n}{2}$,

(e) $\sum_{n=0}^{\infty} \frac{(ni)^n}{n!}$.

5. (**9 pts**) Find the radius of convergence in the following series:

(a) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!} (z - 1 - i)^n$, (b) $\sum_{n=1}^{\infty} \left(\frac{6n+1}{2n+5} \right)^n (z - 2i)^n$,

(c) $\sum_{n=0}^{\infty} \frac{z^{2n}}{4^n}$.

Notice: For Problems 6-8, you need to explicitly write out at least **the first four terms** of the power series as your answer to each question. For example,

$$e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!} = 1 + z + \frac{z^2}{2} + \frac{z^3}{6} + \cdots.$$

6. (**10 pts**) If $0 < |z| < 1$, find the Maclaurin series for

$$\text{(a)}\ \ f(z) = \frac{1}{1+z^2}, \qquad\qquad \text{(b)}\ \ f(z) = \frac{1}{(1+z)^2}.$$

7. (**16 pts**) Find the Laurent series for

$$f(z) = \frac{1}{(z-1)^2(z-3)}$$

(a) in powers of $z-1$ in the domain $0 < |z-1| < 2$, and
(b) in powers of $z-3$ in the domain $|z-3| > 3$, respectively.

8. (**16 pts**) Find the Laurent series for

$$f(z) = \frac{z+1}{z(z-4)^3}$$

in powers of $z-4$. [*Notice that you need to consider all situations for the domains in the whole complex plane.*]