CO2013: Complex Analysis, Quiz-2, Fall 2016

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Notice:
a) *Term grading policy: Quiz-2*$\times 10\%$.
b) *Total* **100 points (2 pages, see the next page for Problems 6-8!)** *in this exam.*
c) *Exam Time*: **10:00AM−11:50AM**, Nov. 14, 2016.

1. (**10 pts**) If $f(z) = 3(z-i)^{-2} + 2(z-i)^{-1} + 1 - 2(z-i) - 3(z-i)^2$, evaluate
$$\oint_C f(z)dz,$$
where C is the circle $|z-i| = 2$ traversed once clockwise.

2. (**20 pts**) Evaluate
(a) $\oint_C \bar{z}dz$, where C is the circle $|z| = 3$ traversed once counterclockwise, and
(b) $\int_\Gamma \mathrm{Re}\{z\}dz$ along the directed line segment from $z = 1$ to $z = 2 + 3i$.

3. (**10 pts**) If C is the circle $|z| = 4$ traversed once, show that
$$\left| \oint_C \frac{dz}{z^2 - 2i} \right| \leq \frac{4\pi}{7}.$$

4. (**10 pts**) Let C be the circle $|z| = 2$ traversed once in the positive sense. Compute
$$\oint_C \frac{5z^3 + 2z + 1}{(z+i)^3} dz.$$

5. (**15 pts**) Compute
$$\int_\Gamma \frac{2z^2 - z + 1}{z^3 + z^2 - z - 1} dz,$$
where Γ is the figure-eight contour traversed once as shown in Fig. 1.

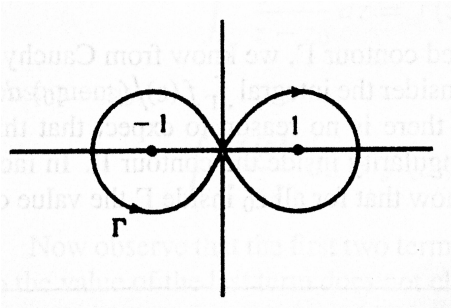

Fig. 1: Problem 5.

6. (**10 pts**) Compute
$$\oint_C \frac{z+i}{z^3+2z^2+z}dz,$$
where C is (a) the circle $|z|=1$ traversed once in the positive sense, and (b) the circle $|z+2-i|=2$ traversed once in the negative sense.

7. (**10 pts**) If f is analytic inside and on the circle $|z-z_0|=r$, prove that
$$f^{(n)}(z_0)=\frac{n!}{2\pi r^n}\int_0^{2\pi} f(z_0+re^{i\theta})e^{-in\theta}d\theta.$$

8. Suppose that f is analytic in $|z|<1$ and that $|f(z)|<1/(1-|z|)$. For a given R, $0<R<1$, prove that

   (a) (**10 pts**) $\left|f^{(n)}(0)\right| \le \frac{n!}{R^n(1-R)}$, and

   (b) (**5 pts**) the upper bound is smallest when $R=n/(n+1)$.