

Notice:

- a) Term grading policy: Final \times **35%**.
- b) Total 120 points in this exam.
- c) Exam Time: 1:00PM–3:00PM, Jan. 11, 2019.

1. (20 pts) Evaluate the integral $\oint_C \frac{f'(z)}{f(z)} dz$ for the following functions:

(a) $f(z) = \frac{(z - 3iz - 2)^2}{z(z^2 - 2z + 2)^5}$, C is $|z| = \frac{3}{2}$,

(b) $f(z) = z^6 - 2iz^4 + (5 - i)z^2 + 10$, C encloses all the zeros of f .

2. (20 pts) Evaluate

$$\text{P.V.} \int_0^{\infty} \frac{dx}{\sqrt{x}(x^2 - 4)}.$$

Notice: You have to write down complete analysis to all parts of the integral (tell the reason even if the value of the part is zero).

3. (80 pts) Evaluate each of the following integrals:

(a) $\text{P.V.} \int_{-\infty}^{\infty} \frac{x \sin(3x)}{x^4 + 4} dx,$

(b) $\text{P.V.} \int_0^{\infty} \frac{\sin(2x)}{x(x^2 + 1)^2} dx,$

(c) $\int_0^{2\pi} \frac{\cos 2\theta}{5 - 4 \sin \theta} d\theta,$

(d) $\text{P.V.} \int_{-\infty}^{\infty} \frac{\sin(3x)}{x - 2i} dx.$