

CO2013: Complex Analysis, Quiz-2, Fall 2018
 Department of Communication Engineering, National Central University
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Notice:

- a) Term grading policy: Quiz-2 \times 15%.
- b) Total 100 points (2 pages, see the next page for Problems 6-8!) in this exam.
- c) Exam Time: 1:00PM–2:50PM, Nov. 16, 2018.

1. (10 pts) If $f(z) = (6z - i)^{-3} + (2z - i)^{-2} - (3z - i)^{-1} + 1$, evaluate

$$\oint_C f(z) dz,$$

where $C : |z - i| = 4$ in the clockwise direction.

2. (15 pts) Let C be the perimeter of the square with vertices at the points $z = 0$, $z = 1$, $z = 1 + i$, and $z = i$ traversed once in that order. Evaluate $\oint_C \bar{z}^2 dz$.
3. (15 pts) Let Γ be the arc of the circle $|z| = e$ that lies in the second quadrant. Find an upper bound of

$$\left| \int_{\Gamma} \text{Log } z \, dz \right|.$$

4. (15 pts) Evaluate $\oint 1/(z^2 + 1) dz$ along the three closed contours Γ_1 , Γ_2 , and Γ_3 in Fig. 1.

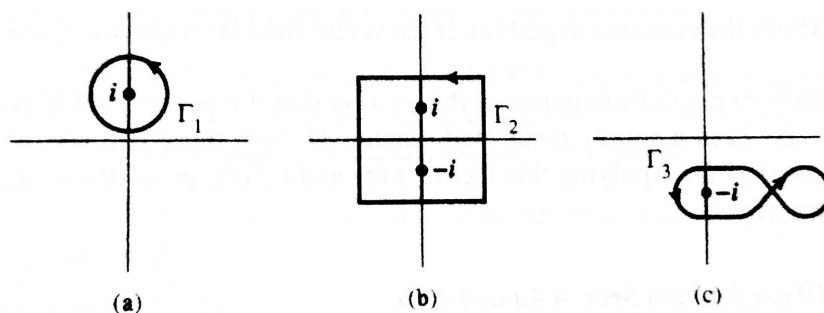


Fig. 1: Problem 4.

5. (15 pts) Evaluate

$$\oint_C \frac{e^{iz}}{(z^2 + 1)^2} dz,$$

where $C : |z| = 3$ in the counterclockwise direction.

6. (10 pts) Let

$$I(R) = \oint_{|z|=R} \frac{zdz}{(z-1)^3},$$

show that (a) $\lim_{R \rightarrow \infty} I(R) = 0$, and (b) $\lim_{R \rightarrow 0} I(R) = 0$.

7. (10 pts) Compute

$$\int_{\Gamma} \frac{\cos z}{z^3(z-3)} dz.$$

along the contour indicated in Fig. 2.

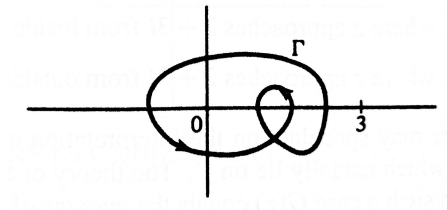


Fig. 2: Problem 7.

8. (10 pts) Let Γ be a simple closed contour and $P(z) = c(z-z_1)(z-z_2)\cdots(z-z_n)$, where z_1, z_2, \dots, z_n all lie inside Γ . Evaluate the integral

$$\frac{1}{2\pi i} \oint_{\Gamma} \frac{P'(z)}{P(z)} dz.$$