CO2013: Complex Analysis, Exam-2, Fall 2019

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Notice:
a) Term grading policy: Exam-2×20%.
b) Total 100 points (2 pages, see the next page for problems 6-7.) in this exam.
c) Exam Time: 1:00PM−2:50PM, Nov. 22, 2019.

1. (10+10+10 pts)
   (a) Show that $\cos^{-1} z = -i \log\left[z + (z^2-1)^{1/2}\right]$.
   (b) Find $\dfrac{d}{dz}\cos^{-1} z$.
   (c) Find the solutions of the equation $\cos z = 2i$.

2. (10 pts) Show that
$$\oint_{|z|=1} \bar{z}dz = \oint_{|z|=1} \frac{1}{z}dz.$$

3. (10 pts) Evaluate
$$\int_C (z^2 - z + 2)dz$$
from i to 1 along the contour C given in Fig. 1.

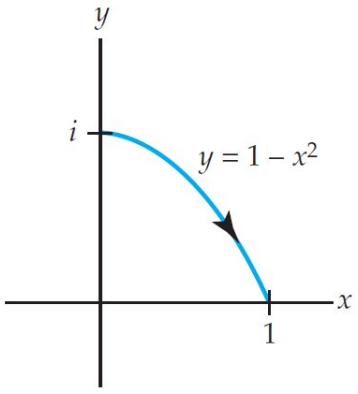

Fig. 1: Problem 3.

4. (10 pts) Show that an upper bound for the absolute value of the integral $\displaystyle\int_C \frac{1}{z^2+1}dz$ is $\dfrac{1}{3\sqrt{10}}$, where C is the line segment from $z = 3$ to $3 + i$.

5. (15 pts) Evaluate $\displaystyle\oint_{C:|z|=1} \frac{1}{z^3 + 2iz^2}dz$.

6. (10 pts) Evaluate $\oint_C \dfrac{8z-3}{z^2-z}dz$, where C is the "figure-eight" contour shown in Fig. 2.

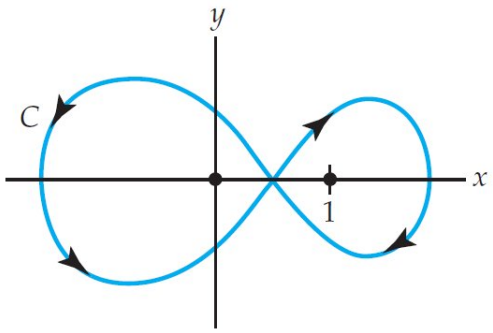

Fig. 2: Problem 6.

7. (15 pts) Evaluate $\oint_C \dfrac{e^{iz}}{(z^2+1)^2}dz$, where C is the "figure-eight" contour shown in Fig. 3.

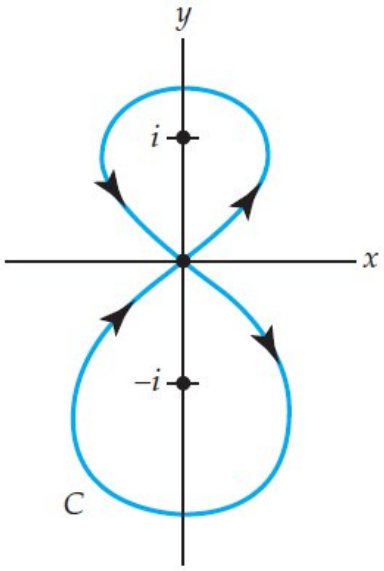

Fig. 3: Problem 7.