

Notice:

- a) Term grading policy: Final  $\times$  **35%**.
- b) Total 120 points in this exam.
- c) Exam Time: 1:00PM–2:50PM, Jan. 3, 2020.

1. (10 pts) Suppose that  $f(z)$  is analytic and has zero of order  $m$  at the point  $z_0$ .

Show that the integral  $\frac{1}{2\pi i} \oint_C \frac{f'(z)}{f(z)} dz = m$  for  $C$  enclosing  $z_0$ .

2. (20 pts) Compute the residue at each singularity of the following functions:

(a)  $f(z) = \frac{\cos z}{z^2(z - \pi)^3}$ ,                      (b)  $\frac{z - 1}{\sin z}$ .

3. (10 pts) Evaluate  $\oint_C e^{1/z} \sin\left(\frac{1}{z}\right) dz$ , where  $C$  is  $|z|=1$ .

4. (80 pts) Evaluate each of the following integrals:

(a) P.V.  $\int_{-\infty}^{\infty} \frac{x \sin x}{x^2 - 2x + 10} dx$ ,                      (b) P.V.  $\int_{-\infty}^{\infty} \frac{\cos x}{(x^2 + 1)(x^2 + 9)} dx$ ,

(c)  $\int_0^{2\pi} \frac{\sin^2 \theta}{5 + 4 \cos \theta} d\theta$ ,                      (d)  $\int_0^{\infty} \frac{dx}{\sqrt{x}(x + 4)}$ .