CO2013: Complex Analysis, Exam-1, Fall 2019

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Notice:
a) Term grading policy: Exam-1×20%.
b) Total 100 points in this exam.
c) Exam Time: 1:00PM−2:50PM, 18 Oct., 2019.

1. (10 pts) Prove that $|z_1 - z_2|^2 = |z_1|^2 - 2\text{Re}(z_1\bar{z}_2) + |z_2|^2$.

2. (10+10 pts)
   (a) Using the De Moivre's formula to establish the identities for $\cos^4\theta$ in terms of $\cos 2\theta$ and $\cos 4\theta$.
   (b) Compute the integral
   $$\int_0^{2\pi} \cos^4\theta d\theta.$$

3. (5+5 pts) Find all the values of the following:
   (a) $(-16)^{1/4}$, (b) $(\frac{2i}{1+i})^{1/6}$.

4. (10 pts) Sove the equations
   $$\frac{z^2 - 3z + 1}{3 - 2z} = i$$

5. (15 pts) Let
   $$f(z) = \frac{x^2 + iy^2}{|z|^2}$$
   when $z \neq 0$, and let $f(0) = 1$. Show that $f(z)$ is not continuous at $z_0 = 0$

6. (15 pts) Construct an analytic function whose real part is $u(x, y) = x^3 - 3xy^2 + y$.

7. (20 pts) Show that the function $f(z) = e^{x^2-y^2}[\cos(2xy) + i\sin(2xy)]$ is entire, and find its derivative.