CO2013: Complex Analysis, Exam-2, Fall 2020

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Notice:
a) Term grading policy: Exam-2×25%.
b) Total 100 points (2 pages, see the next page for problem 7.) in this exam.
c) Exam Time: 1:00PM−2:50PM, Nov. 20, 2020.

1. (10 pts) Solve the equation $\text{Log}(z^2-1) = \frac{\pi i}{2}$ with the solution in the format of $x+iy$, where x and y are real values.

2. (15 pts) Express the following complex numbers in the format of $x+iy$, where x and y are real values:
(a) $2^{\pi i}$ (b) $(1-i)^{1+i}$ (c) $(-1)^{2/3}$

3. (5+10 pts) Let $f(z) = 1/(1-z)^2$.
(a) Find the formula of $f^{(n)}(z)$ and also show $f^{(n)}(0) = (n+1)!$.
(b) Let $0 < R < 1$. Using the ML-inequality, show that
$$(n+1)! \le \frac{n!}{R^n(1-R)^2}.$$

4. (15 pts) Let $f(z)$ be analytic on and inside a simple closed contour C, and z_1, z_2, and z_3 lie inside C. Show that
$$\frac{1}{2\pi i}\oint_C \frac{f(z)}{(z-z_1)(z-z_2)(z-z_3)}dz = \frac{f(z_1)}{(z_1-z_2)(z_1-z_3)} + \frac{f(z_2)}{(z_2-z_1)(z_2-z_3)} + \frac{f(z_3)}{(z_3-z_1)(z_3-z_2)}.$$

5. (15 pts) Evaluate $\oint_C \frac{\cos z}{(z-1)^3(z-5)^2}dz$, where C is the circle $|z-4| = 2$.

6. (15 pts) Let $z = x+iy$. Compute $\int_C (|z-1+i|^2 - z)dz$ along the semicircle in the counterclockwise direction as shown in Fig. 1.

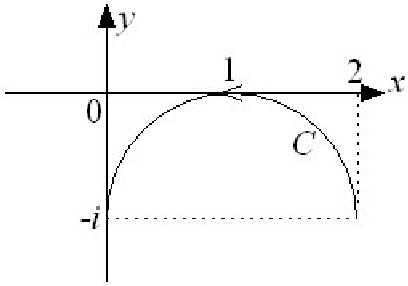

Fig. 1: Problem 6.

7. (15 pts) Evaluate $\oint_C \frac{1}{z^4 - 1} dz$, where C is the "four-leaf clover" path as shown in Fig. 2.

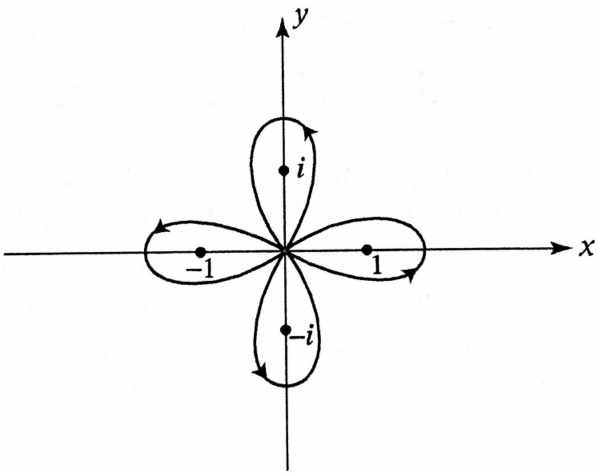

Fig. 2: Problem 7.