CO2013: Complex Analysis, Exam-1, Fall 2020

Department of Communication Engineering, National Central University

Prof. Dah-Chung Chang (office: E1-311, e-mail: dcchang@ce.ncu.edu.tw)

Notice:
a) Term grading policy: Exam-1×15%.
b) Total 100 points in this exam.
c) Exam Time: 1:00PM−2:50PM, 23 Oct., 2020.

1. (10 pts) Prove that $\left(\dfrac{1+i\tan\theta}{1-i\tan\theta}\right)^n = \dfrac{1+i\tan n\theta}{1-i\tan n\theta}$, where n is any integer.

2. (10 pts) Show that $f(z) = |z|$ is nowhere differentiable.

3. (15 pts) Find all solutions of the equation $z^6 + z^3 + 1 = 0$ in the form of $x + iy$. ($\sin 20^o = 0.342$, $\sin 40^o = 0.643$, $\sin 80^o = 0.985$, $\cos 20^o = 0.94$, $\cos 40^o = 0.766$, $\cos 80^o = 0.174$)

4. (15 pts) Show that the function $f(z) = (x^2+y)+i(y^2-x)$ is nowhere analytic.

5. (15 pts) Show that if v is a harmonic conjugate of u in a domain D, then uv is harmonic in D.

6. (15 pts) Construct an analytic function $f(z)$ in terms of z, whose real part is
$$u(x,y) = \frac{y}{x^2+y^2},$$
where $z = x + iy$.

7. (20 pts) Suppose that $u(r,\theta) = r^2\sin(2\theta)$, construct an analytic function $f(z) = u(r,\theta) + iv(r,\theta)$ in terms of z, where $z = re^{i\theta}$.