CO2013: Complex Analysis, Exam-3, Fall 2021

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Notice:
a) Term grading policy: Exam-3×25%.
b) Total 100 points in this exam.
c) Exam Time: 1:00PM−2:50PM, Dec. 16, 2021.

1. (10 pts) Show that the following series converge.

   (a) $\sum_{n=0}^{\infty} \frac{n^2}{4^n}$,

   (b) The sequence $\{z_n\}_1^{\infty}$ converges if and only if the series $\sum_{n=1}^{\infty}(z_{n+1} - z_n)$ converges.

2. (10 pts) Find the circle and radius of convergence of the following series:

   (a) $\sum_{n=0}^{\infty} \frac{(z-2-i)^{2n}}{2^{3n}}$, (b) $\sum_{n=0}^{\infty} \frac{1}{n}\left(\frac{i}{1+i}\right)(z-i)^n$.

3. (15 pts) Compute $\oint_{C:|z|=1} \frac{e^{z/2}\sin z}{z^5} dz$.

4. (30 pts) Find the Taylor series centered at the indicated point z_0 for the given function and give the radius of convergence.

   (a) $(z-1)e^{-3z}, z_0 = 1$ (b) $\frac{z+1}{z(z-2)^2}, z_0 = 0$

5. (20 pts) Find the Laurent series for $f(z) = \frac{1}{z+z^2}$ in the regions of (a) $1 < |z|$ (b) $0 < |z+1| < 1$.

6. (15 pts) Find the Laurent series for $f(z) = \frac{1}{(z-2)(z-3)}$ in the region of $1 < |z-1| < 2$.