CO2013: Complex Analysis, Exam-2, Fall 2022

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Notice:
a) Term grading policy: Exam-2×30%.
b) Total 100 points in this exam.
c) Exam Time: 1:00PM−2:50PM, Dec. 8, 2022.

1. (10 pts) Evaluate $\oint_C (z^2 + e^{\bar{z}} + \mathrm{Re}(z))dz$, where C is the triangle with vertices $z = 0$, $z = 1 + 2i$, and $z = 1$.

2. (20 pts) Evaluate the given integral along the indicated closed contour(s):
   (a) $\oint_C \frac{e^{-z}\sin z}{z^3} dz$ for $C : |z - 1| = 3$,
   (b) $\oint_C \frac{z^2 + 3z + 2i}{z^2 + 3z - 4} dz$ for (i) $C : |z| = 2$, (ii) $C : |z + 5| = 3$.

3. (20 pts) Find the circle of convergence of the following series:
   (a) $\sum_{k=0}^{\infty} \frac{4^k}{3k + 7}(z - 2 + 3i)^{2k}$,
   (b) the Taylor series of $f(z) = \csc z$ centered at $z_0 = 3 + 2i$.

4. (15 pts) Find the Maclaurin series and give the radius of convergence for the function $f(z) = \frac{z - 7}{z^2 - 2z - 3}$.

5. (15 pts) The error function $\mathrm{erf}(z)$ is defined by the integral $\mathrm{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt$. Find the Maclaurin series for $\mathrm{erf}(z)$.

6. (20 pts) Find the Laurent series for the function $f(z) = \frac{7z - 3}{z^2 - z}$ centered at $z_0 = 1$.