CO2013: Complex Analysis, Exam-1, Fall 2022

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Notice:
a) Term grading policy: Exam-1×30%.
b) Total 100 points in this exam.
c) Exam Time: 1:00PM−2:50PM, Oct. 13, 2022.

1. (10 pts) Find the following complex numbers in the form of $x + iy$:
   (a) $\log(\sqrt{3} + i)$, (b) $(1 + i)^{3+4i}$.

2. (10 pts) Suppose $f(z) = u + iv$ is analytic. Under what conditions will $g(z) = u - iv$ be also analytic?

3. (15 pts) Find the harmonic conjugate of $e^x \cos y + e^y \cos x + xy$.

4. (15 pts) Let z_1 and z_2 be two complex numbers corresponding to their modului p and q, repectively. As shown in Fig. 1, z_1 and z_2 are lying along two adjacent sides of the parallelogram, where m and n are the lengths of two diagonals. Show that $m^2 + n^2 = 2(p^2 + q^2)$.

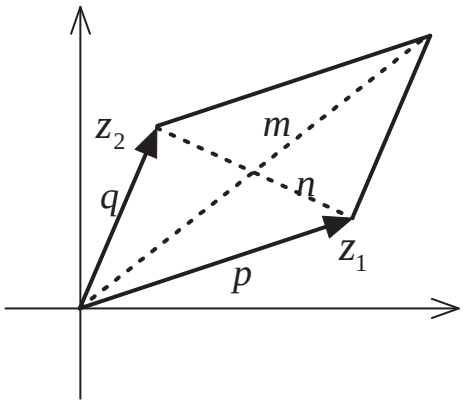

Fig. 1: Problem 4.

5. (15 pts) Prove that

$$\cos\left(\frac{2\pi}{n}\right) + \cos\left(\frac{4\pi}{n}\right) + \cdots + \cos\left[\frac{2(n-1)\pi}{n}\right] = -1.$$

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6. (15 pts) Let $z = x + iy$, where $x, y \in \mathbb{R}$. Define that

$$\sinh y = \frac{e^y - e^{-y}}{2} \qquad \text{and} \qquad \cosh y = \frac{e^y + e^{-y}}{2}.$$

Prove that $|\cosh z|^2 = \cosh^2 x - \sin^2 y$.

7. (20 pts) (a) Show that $\cosh^{-1} z = \log(z + (z^2 - 1)^{1/2})$, and (b) find all solutions to the equation $\cosh^2 z = -1$.