

Notice:

- a) Term grading policy: Exam-3 (Final)  $\times 40\%$ .
- b) Total 115 points in this exam.
- c) Exam Time: 1:00PM–3:00PM, 11 Jan., 2024.

1. (25 pts) Evaluate the following integral:

$$\text{P.V.} \int_0^{\infty} \frac{x^2 dx}{(x^2 + 4)(x^2 + 1)^2}.$$

2. (25 pts) Derive the following integration formula with two real values  $a$  and  $b$ , and  $a > b > 0$ :

$$\text{P.V.} \int_0^{\infty} \frac{\sqrt[3]{x}}{(x+a)(x+b)} dx = \frac{2\pi}{\sqrt{3}} \cdot \frac{\sqrt[3]{a} - \sqrt[3]{b}}{a-b}.$$

3. (25 pts) Use residues to find the inverse Laplace transform in the following:

$$F(s) = \frac{2s - 2}{(s + 1)(s^2 + 2s + 5)}.$$

4. (40 pts) Use residues to find the Fourier transform of  $f(t) = \frac{t}{t^4 + 1}$ .