CO2013: Complex Analysis, Exam-1, Fall 2023

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Notice:
a) Term grading policy: Exam-1×30%.
b) Total 100 points in this exam.
c) Exam Time: 1:00PM−2:50PM, Oct. 26, 2023.

1. (10 pts) Find the following complex numbers in the form of $x + iy$:
   (a) $\cot(\pi + 2i)$, (b) $(1 + \sqrt{3}i)^{3i}$.

2. (20 pts) Find all complex values z satisfying the following equations:
   (a) $\sin z = \cos z$, (b) $\cos z = i \sin z$.

3. (20 pts)
   (a) Show that $\sin^{-1} z = -i \log[iz + (1 - z^2)^{1/2}]$. (5 pts)
   (b) Show that $\sin^{-1} z + \cos^{-1} z = (4n + 1)\pi/2, n = 0, \pm 1, \pm 2, \cdots$. (15 pts)

4. (20 pts) The function $f(z) = |z|^2$ is continuous at the origin.
   (a) Show that f is differentiable at the origin.
   (b) Show that f is not differentiable at any point $z \neq 0$.

5. (15 pts) Show that if f is analytic in a domain D, and $f'(z) = 0$, then f is a constant.

6. (15 pts) Suppose $f(z) = u(r, \theta) + iv(r, \theta)$ is analytic in a domain D not containing the origin. Find v, a harmonic conjugate of u, for the given function

$$u(r, \theta) = \frac{10r^2 - \sin 2\theta}{r^2}.$$