CO2013: Complex Analysis, Exam-2, Fall 2023

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Notice:
a) Term grading policy: Exam-2×30%.
b) Total 100 points in this exam.
c) Exam Time: 1:00PM−2:50PM, Dec. 7, 2023.

1. (20 pts) Evaluate the contour integral $\oint_C \left(\frac{1}{(z+1)^3} - \frac{5}{z+i} + 8 \right) dz$, where C is the circle $|z+i| = 1$.

2. (20 pts) Evaluate the given integral along the indicated closed contour(s):
   (a) $\oint_C \frac{dz}{z^2(z^2+1)}$ for $C : |z-i| = \frac{3}{2}$,
   (b) $\oint_C \frac{e^{-z}\sin z}{z^3} dz$ for $C : |z-1| = 3$.

3. (20 pts) Prove the following:
   (a) $\sum_{n=0}^{\infty} (n+1)^2 z^n = \frac{1+z}{(1-z)^3}$, and what value of z is this valid?
   (b) Suppose that $\sum_{n=0}^{\infty} c_n z^n$ has radius of convergence R, then $\sum_{n=0}^{\infty} c_n^2 z^n$ has radius of convergence R^2.

4. (10 pts) For $|z-i| < \sqrt{2}$, find the power series representation of $\frac{1}{1-z}$.

5. (10 pts)
   (a) Prove that $4\cos^3 z = \cos 3z + 3\cos z$.
   (b) Find the Maclaurin series for $\cos^3 z$.

6. (20 pts) Find the Laurent series for $(z^2 - 5z + 6)^{-1}$ at $z = 0$.