CO2013: Complex Analysis, Exam-2, Fall 2024

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Notice:

- a) Term grading policy: Exam-2×30%.
- b) Total 100 points in this exam.
- c) Exam Time: 1:00PM-2:50PM, Dec. 12, 2024.
- 1. (15 pts) Compute the integral $\oint_C \frac{2z+1}{z(z-1)^2} dz$ along the figure-eight contour C sketched in Fig.1.

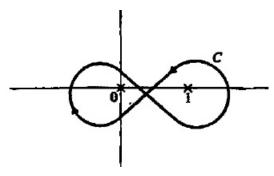


Fig. 1: Problem 1.

2. (15 pts) Let C be the ellipse $x^2/4 + y^2/9 = 1$, and define

$$G(z) = \oint_C \frac{\zeta^2 - \zeta + 2}{\zeta - z} d\zeta$$

where z is inside C. Find G(1), G'(i), and G''(-i).

- 3. (20 pts) Evaluate $\oint_C \operatorname{Re}(z)dz$, where C is the circle |z|=1.
- 4. (20 pts) Find the circle of convergence of the given power series:

(a)
$$\sum_{k=0}^{\infty} (-1)^k \left(\frac{1+2i}{2}\right)^k (z+2i)^k$$
, (b) $\sum_{k=0}^{\infty} \frac{k!}{(2k)^k} z^{3k}$.

(b)
$$\sum_{k=0}^{\infty} \frac{k!}{(2k)^k} z^{3k}$$
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- 5. (10 pts) Find the first three nonzero terms of the Maclaurin series of $e^{1/(1+z)}$.
- 6. (20 pts) Expand $f(z) = \frac{7z-3}{z^2-z}$ in a Laurent series valid for the given domain: (a) 0 < |z| < 1, (b) 0 < |z-1| < 1.