Notice:
a) Term grading policy: Exam-2×30%.
b) Total 100 points in this exam.
c) Exam Time: 1:00PM−2:50PM, Dec. 12, 2024.

1. (15 pts) Compute the integral $\oint_C \dfrac{2z+1}{z(z-1)^2}dz$ along the figure-eight contour C sketched in Fig.1.
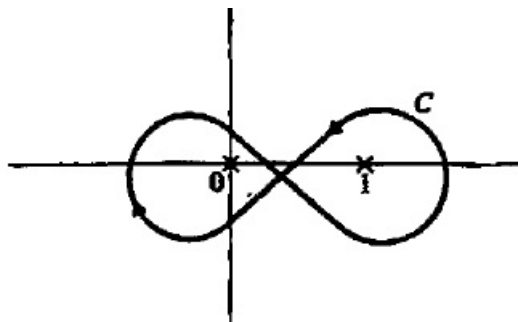

Fig. 1: Problem 1.

2. (15 pts) Let C be the ellipse $x^2/4 + y^2/9 = 1$, and define

$$G(z) = \oint_C \frac{\zeta^2 - \zeta + 2}{\zeta - z}d\zeta$$

where z is inside C. Find $G(1)$, $G'(i)$, and $G''(-i)$.

3. (20 pts) Evaluate $\oint_C \mathrm{Re}(z)dz$, where C is the circle $|z| = 1$.

4. (20 pts) Find the circle of convergence of the given power series:

(a) $\displaystyle\sum_{k=0}^{\infty}(-1)^k \left(\frac{1+2i}{2}\right)^k (z+2i)^k$,
    (b) $\displaystyle\sum_{k=0}^{\infty}\frac{k!}{(2k)^k}z^{3k}$.

5. (10 pts) Find the first three nonzero terms of the Maclaurin series of $e^{1/(1+z)}$.

6. (20 pts) Expand $f(z) = \dfrac{7z-3}{z^2-z}$ in a Laurent series valid for the given domain:
(a) $0 < |z| < 1$,   (b) $0 < |z-1| < 1$.