

CO2013: Complex Analysis, Exam-1, Fall 2024
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Notice:

- a) Term grading policy: Exam-1 \times 30%.
- b) Total 100 points in this exam.
- c) Exam Time: 1:00PM–2:50PM, Oct. 24, 2024.

1. (20 pts) Solve the following equations:

(a) $\text{Log}(z^2 - 1) = \frac{i\pi}{2}$, (b) $e^{2z} + e^z + 1 = 0$.

2. (15 pts) Prove the identity:

$$\sec^{-1} z = -i \log \left[\frac{1}{z} + \left(\frac{1}{z^2} - 1 \right)^{\frac{1}{2}} \right].$$

3. (20 pts)

- (a) Find the identity $\tan^{-1} z$ in the form of logarithmic function of z .
- (b) Consider the principal branch and derive the following identity

$$\frac{d}{dz}(\tan^{-1} z) = \frac{1}{1 + z^2}, \quad z \neq \pm i.$$

4. (20 pts) Find the harmonic conjugate for $u(x, y) = ax^2 + bxy + cy^2$.

5. (15 pts) Show that $f(z) = |z|$ is nowhere differentiable.

6. (10 pts) Find all complex values:

(a) $\left(\frac{2i}{1+i} \right)^{1/6}$, (b) $1^{1/5}$.