CO2013: Complex Analysis, Unit-2 Testbank

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<2016u2>

1. <2016u2> If $f(z) = 3(z-i)^{-2} + 2(z-i)^{-1} + 1 - 2(z-i) - 3(z-i)^2$, evaluate
$$\oint_C f(z)dz,$$
where C is the circle $|z-i| = 2$ traversed once clockwise.

2. <2016u2> Evaluate
(a) $\oint_C \bar{z}dz$, where C is the circle $|z| = 3$ traversed once counterclockwise, and
(b) $\int_\Gamma \text{Re}\{z\}dz$ along the directed line segment from $z = 1$ to $z = 2 + 3i$.

3. <2016u2> If C is the circle $|z| = 4$ traversed once, show that
$$\left| \oint_C \frac{dz}{z^2 - 2i} \right| \leq \frac{4\pi}{7}.$$

4. <2016u2> Let C be the circle $|z| = 2$ traversed once in the positive sense. Compute
$$\oint_C \frac{5z^3 + 2z + 1}{(z+i)^3} dz.$$

5. <2016u2> Compute
$$\int_\Gamma \frac{2z^2 - z + 1}{z^3 + z^2 - z - 1} dz,$$
where Γ is the figure-eight contour traversed once as shown in Fig. 1.

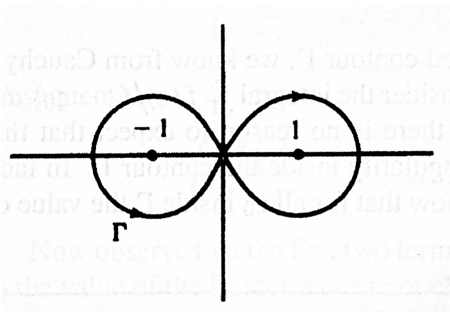

Fig. 1: Problem 5.

6. <2016u2> Compute
$$\oint_C \frac{z+i}{z^3 + 2z^2 + z} dz,$$

where C is (a) the circle $|z| = 1$ traversed once in the positive sense, and (b) the circle $|z + 2 - i| = 2$ traversed once in the negative sense.

7. <2016u2> If f is analytic inside and on the circle $|z - z_0| = r$, prove that

$$f^{(n)}(z_0) = \frac{n!}{2\pi r^n} \int_0^{2\pi} f(z_0 + re^{i\theta})e^{-in\theta} d\theta.$$

8. <2016u2> Suppose that f is analytic in $|z| < 1$ and that $|f(z)| < 1/(1 - |z|)$. For a given R, $0 < R < 1$, prove that

   (a) $\left|f^{(n)}(0)\right| \leq \dfrac{n!}{R^n(1-R)}$, and

   (b) the upper bound is smallest when $R = n/(n + 1)$.

9. <2016u2> If $f(z) = \sum_{n=0}^{\infty} \frac{n^4}{4^n} z^n$ and C is the circle $|z| = 1$ traversed once in the positive sense, compute

$$\oint_C \frac{(z + 1)f(z)}{z^5} dz.$$

<2018u2>

1. <2018u2> If $f(z) = (6z - i)^{-3} + (2z - i)^{-2} - (3z - i)^{-1} + 1$, evaluate

$$\oint_C f(z)dz,$$

where $C : |z - i| = 4$ in the clockwise direction.

2. <2018u2> Let C be the perimeter of the square with vertices at the points $z = 0$, $z = 1$, $z = 1 + i$, and $z = i$ traversed once in that order. Evaluate $\oint_C \bar{z}^2 dz$.

3. <2018u2> Let Γ be the arc of the circle $|z| = e$ that lies in the second quadrant. Find an upper bound of

$$\left| \int_\Gamma \operatorname{Log} z \; dz \right|.$$

4. <2018u2> Evaluate $\oint 1/(z^2 + 1)dz$ along the three closed contours Γ_1, Γ_2, and Γ_3 in Fig. 2.

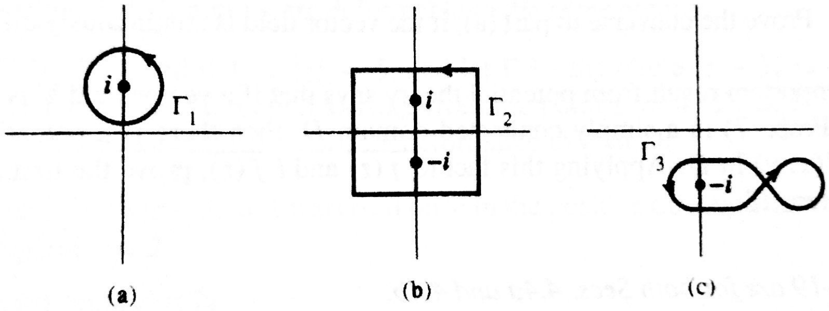

Fig. 2: Problem 4.

5. <2018u2> Evaluate
$$\oint_C \frac{e^{iz}}{(z^2+1)^2}dz,$$
where $C : |z| = 3$ in the counterclockwise direction.

6. <2018u2> Let
$$I(R) = \oint_{|z|=R} \frac{zdz}{(z-1)^3},$$
show that (a) $\lim_{R\to\infty} I(R) = 0$, and (b) $\lim_{R\to 0} I(R) = 0$.

7. <2018u2> Compute
$$\int_\Gamma \frac{\cos z}{z^3(z-3)}dz.$$
along the contour indicated in Fig. 3.

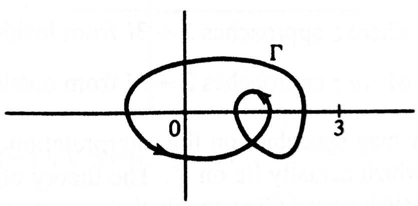

Fig. 3: Problem 7.

8. <2018u2> Let Γ be a simple closed contour and $P(z) = c(z-z_1)(z-z_2)\cdots(z-z_n)$, where $z_1, z_2, \cdots, z_n$ all lie inside Γ. Evaluate the integral
$$\frac{1}{2\pi i}\oint_\Gamma \frac{P'(z)}{P(z)}dz.$$

9. <2018u2> Evaluate the integral
$$\oint_C \frac{3z+1}{z(z-2)^3}dz,$$

where C is the figure-eight contour depicted in Fig. 4.

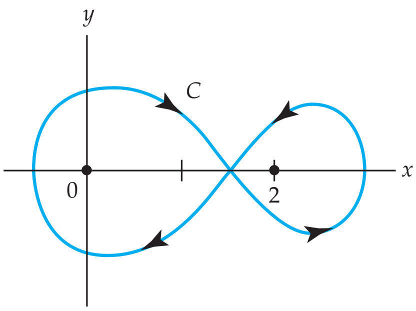

Fig. 4: Problem 9.

<2019u2>

1. <2019u2> Show that
$$\oint_{|z|=1} \bar{z}dz = \oint_{|z|=1} \frac{1}{z}dz.$$

2. <2019u2> Evaluate
$$\int_C (z^2 - z + 2)dz$$
from i to 1 along the contour C given in Fig. 5.

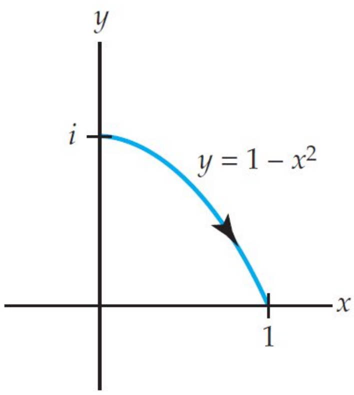

Fig. 5: Problem 2.

3. <2019u2> Show that an upper bound for the absolute value of the integral $\int_C \frac{1}{z^2+1}dz$ is $\frac{1}{3\sqrt{10}}$, where C is the line segment from $z = 3$ to $3 + i$.

4. <2019u2> Evaluate $\oint_{C:|z|=1} \frac{1}{z^3 + 2iz^2} dz$.

5. <2019u2> Evaluate $\oint_C \frac{8z-3}{z^2-z} dz$, where C is the "figure-eight" contour shown in Fig. 6.

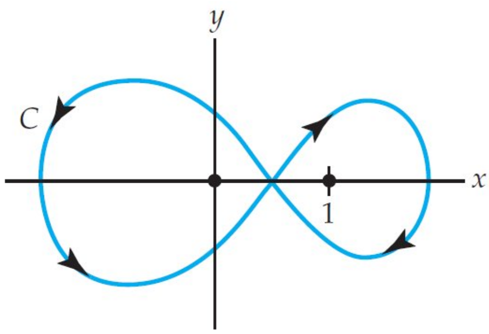

Fig. 6: Problem 5.

6. <2019u2> Evaluate $\oint_C \frac{e^{iz}}{(z^2+1)^2} dz$, where C is the "figure-eight" contour shown in Fig. 7.

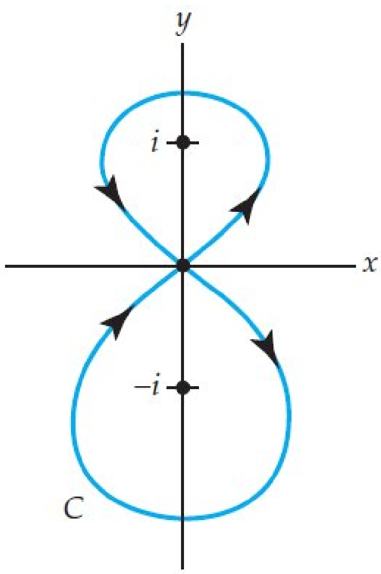

Fig. 7: Problem 6.

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1. <2020u2> Let $f(z) = 1/(1-z)^2$.
   (a) Find the formula of $f^{(n)}(z)$ and also show $f^{(n)}(0) = (n+1)!$.
   (b) Let $0 < R < 1$. Using the ML-inequality, show that

$$(n+1)! \le \frac{n!}{R^n(1-R)^2}.$$

2. <2020u2> Let $f(z)$ be analytic on and inside a simple closed contour C, and z_1, z_2, and z_3 lie inside C. Show that

$$\frac{1}{2\pi i}\oint_C \frac{f(z)}{(z-z_1)(z-z_2)(z-z_3)}dz = \frac{f(z_1)}{(z_1-z_2)(z_1-z_3)} + \frac{f(z_2)}{(z_2-z_1)(z_2-z_3)} + \frac{f(z_3)}{(z_3-z_1)(z_3-z_2)}.$$

3. <2020u2> Evaluate $\displaystyle\oint_C \frac{\cos z}{(z-1)^3(z-5)^2}dz$, where C is the circle $|z-4|=2$.

4. <2020u2> Let $z = x+iy$. Compute $\displaystyle\int_C (|z-1+i|^2 - z)dz$ along the semicircle in the counterclockwise direction as shown in Fig. 8.

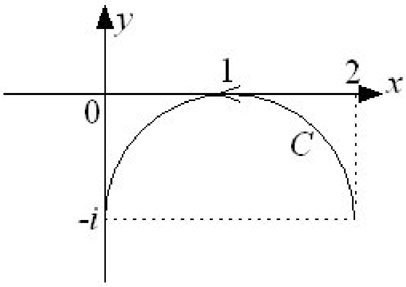

Fig. 8: Problem 4.

5. <2020u2> Evaluate $\displaystyle\oint_C \frac{1}{z^4-1}dz$, where C is the "four-leaf clover" path as shown in Fig. 9.

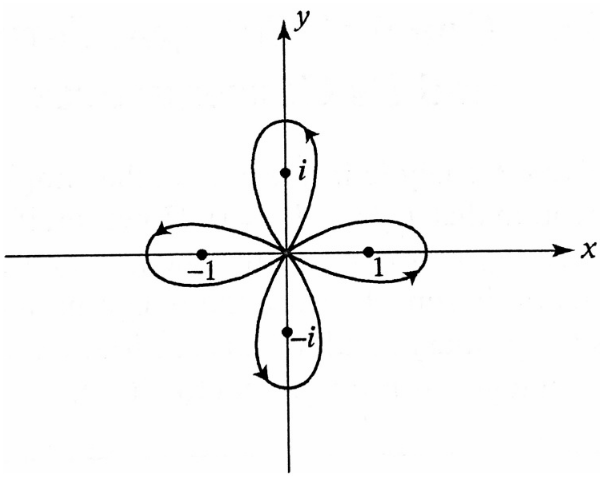

Fig. 9: Problem 5.