CO2013: Complex Analysis, Unit-3 Testbank

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<2016u3>

1. <2016u3> Explain and determine if the given series converges or diverges in the following:

   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{i^n}{(1+i)^{n-1}}$, (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(1+i)^n}{(-1)^n n^3}$,

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{i^n}{n^2}$, (d) $\displaystyle\sum_{n=0}^{\infty} \frac{i^n}{2}$,

   (e) $\displaystyle\sum_{n=0}^{\infty} \frac{(ni)^n}{n!}$.

2. <2016u3> Find the radius of convergence in the following series:

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!}(z-1-i)^n$, (b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{6n+1}{2n+5}\right)^n (z-2i)^n$,

   (c) $\displaystyle\sum_{n=0}^{\infty} \frac{z^{2n}}{4^n}$.

3. <2016u3> If $0 < |z| < 1$, find the Maclaurin series for

   (a) $f(z) = \dfrac{1}{1+z^2}$, (b) $f(z) = \dfrac{1}{(1+z)^2}$.

4. <2016u3> Find the Laurent series for

$$f(z) = \frac{1}{(z-1)^2(z-3)}$$

   (a) in powers of $z-1$ in the domain $0 < |z-1| < 2$, and
   (b) in powers of $z-3$ in the domain $|z-3| > 3$, respectively.

5. <2016u3> Find the Laurent series for

$$f(z) = \frac{z+1}{z(z-4)^3}$$

   in powers of $z-4$. [Notice that you need to consider all situations for the domains in the whole complex plane.]

<2018u3>

1. <2018u3> Find the radius of convergence in the following series:

$$\text{(a)} \sum_{n=1}^{\infty} \frac{(z+3i)^n}{n^2(3+4i)^n}, \qquad \text{(b)} \sum_{n=1}^{\infty} \frac{z^n}{n^n}, \qquad \text{(c)} \sum_{n=0}^{\infty} \frac{n!}{(2n)^n} z^{3n}.$$

2. <2018u3> Find the region in the complex plane for which $\sum_{n=0}^{\infty} (\frac{z-1}{z+2})^n$ converges.

3. <2018u3>
   (a) Find the Taylor series of $\sin z$ and $\cos z$, and use them to write out the first three terms of the power series of $\cot z$ in powers of z.
   (b) Using the result obtained from (a), calculate

$$\oint_C \frac{\cot z}{z^2} dz,$$

   where $C : |z| = 1$.

4. <2018u3> Find the Laurent series for

$$f(z) = \frac{z+1}{(z-1)(z-4)^3}$$

   with center 4 in the domain $|z-4| > 3$.

5. <2018u3> Find the Laurent series for

$$f(z) = \frac{z}{(z-1)(z-3)}$$

   with center 1 in the domain $0 < |z-1| < 2$.

6. <2018u3> Find the power series representation for

$$f(z) = \frac{3}{2+z-z^2}$$

   in powers of $z+1$. [Notice that you need to consider all situations for the domains in the whole complex plane.]

<2019u3>

1. <2019u3> Find the radius of convergence in the following series:

$$\text{(a)} \sum_{n=1}^{\infty} \frac{(-1)^n}{n2^n}(z-1-i)^n, \qquad \text{(b)} \sum_{n=0}^{\infty} \frac{(z-4-3i)^n}{5^{2n}},$$

$$\text{(c)} \sum_{n=0}^{\infty} \frac{(2n)!}{(n+2)(n!)^2}(z-i)^{2n}.$$

2. <2019u3> Determine whether the given sequence converges or diverges:

(a) $\left\{\dfrac{n(1+i^n)}{n+1}\right\}$, (b) $\left\{e^{1/n}+2(\tan^{-1} n)i\right\}$.

3. <2019u3> Let $f(z)=\sum_{n=0}^{\infty}(n^3/3^n)z^n$. Compute the following

(a) $\oint_{|z|=1}\dfrac{f(z)}{z^4}dz$, (b) $\oint_{|z|=1}\dfrac{f(z)\sin z}{z^3}dz$.

4. <2019u3> Suppose the function $f(z)=\dfrac{3-i}{1-i+z}$ is expanded in a Taylor series with center $4-2i$. What is the radius of convergence?

5. <2019u3> Find the first three nonzero terms in Maclaurin expansion of

$$f(z)=\int_0^z e^{\tau^3}d\tau.$$

6. <2019u3> Expand $f(z)=\dfrac{z}{(1-z)^3}$ in a Maclaurin series and give the radius of convergence.

7. <2019u3> Expand $f(z)=\dfrac{1+z}{1-z}$ in a Taylor series centered at $z_0=i$ and give the radius of convergence.

8. <2019u3> Expand the following functions in Laurent series:
(a) $f(z)=\dfrac{1}{(z-1)^2(z-3)}$ for $0<|z-1|<2$, and
(b) $f(z)=\dfrac{1}{z(z-1)}$ for $1<|z-2|<2$.

<2020u3>

1. <2020u3> Explain whether the following series converge or diverge:

(a) $\sum_{n=1}^{\infty}\left(i^n-\dfrac{1}{n^2}\right)$, (b) $\sum_{n=1}^{\infty}\dfrac{n!}{n^n}$.

2. <2020u3> Find the domain in which convergence holds for each of the following series of functions:

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{z^n}{n!}$, (b) $\displaystyle\sum_{n=0}^{\infty} (z+5i)^{2n}(n+1)^2$,

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{z^n}{z^n - 3^n}$.

3. <2020u3> Determine the radius of convergence of the Taylor series of the function $f(z) = \dfrac{4+5z}{1+z^2}$ at the center point $z_0 = 2+5i$.

4. <2020u3> Determine the annulus of convergence of the Laurent series $\displaystyle\sum_{n=-\infty}^{\infty} \frac{z^n}{2^{|n|}}$.

5. <2020u3> Assume that $f(z)$ is analytic at the origin and that $f(0) = f'(0) = 0$. Prove that $f(z)$ can be written in the form $f(z) = z^2 g(z)$, where $g(z)$ is analytic at $z = 0$.

6. <2020u3> Find the Taylor expansions of the following functions with center z_0:

(a) $\dfrac{1}{1-z}$, $z_0 = i$, (b) $\mathrm{Log}\left(\dfrac{1-z}{1+z}\right)$, $z_0 = 0$.

7. <2020u3> Expand $f(z) = \dfrac{z}{(z+1)^2(z-2)}$ in a Maclaurin series and give the radius of convergence.

8. <2020u3> Find the Laurent series for $f(z) = \dfrac{1}{(z-2)(z-1)^3}$ in the region of $0 < |z-2| < 1$.