CO2013: Complex Analysis, Unit-4 Testbank

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<2016u4>

1. <2016u4> What is the order of the pole of
$$f(z) = \frac{1}{(6\sin z - 1 + z^3)^2}.$$

2. <2016u4> Suppose that $f(z)$ is analytic and has a zero of order m at $z = z_0$. Let $C : |z - z_0| = 1$ in positively oriented direction. Evaluate
$$\oint_C \frac{f'(z)}{f(z)} dz.$$

3. <2016u4> Evaluate each of the following contour integrals with positive orientation:

   (a) $\oint_{|z|=2} \tan z dz$, (b) $\oint_{|z|=1} \frac{dz}{z\sin z}$,

   (c) $\oint_{|z+1-i|=1} \frac{dz}{z^2+z+1}$, (d) $\oint_{|z|=1} z^2 \sin\left(\frac{1}{2z}\right) dz$.

4. <2016u4> Evaluate each of the following integrals:

   (a) $\int_{-\pi}^{\pi} \frac{d\theta}{1+\sin^2\theta}$, (b) $\int_0^{2\pi} e^{\cos\theta}\cos(n\theta - \sin\theta)d\theta$,

   (c) P.V. $\int_{-\infty}^{\infty} \frac{\cos 2x}{x-3i} dx$, (d) $\int_0^{\infty} \frac{\cos x}{x^4-1} dx$.

5. <2016u4> What is the order of the pole of
$$f(z) = \frac{1}{(\sin z - z + z^2)^2}$$
at $z = 0$?

6. <2016u4> Let $f(z) = \dfrac{z^2(z+i)^4(z-3)^6 e^{z^2}}{3(z-1)^3(2z-7)^5}$, evaluate $\oint_{D_2^+(0)} \frac{f'(z)}{f(z)} dz$.

7. <2016u4> Evaluate each of the following contour integrals with positive orientation:

   (a) $\oint_{|z|=1} e^{1/z}\tan\left(\frac{1}{2z}\right) dz$, (b) $\oint_{|z|=1} \frac{e^z}{z^3+5z^2} dz$.

8. <2016u4> Evaluate

$$\int_0^\infty \frac{\log x}{(x^2+1)^2} dx.$$

9. <2016u4> Show that $-1 < \alpha < 1$,

$$\text{P.V.} \int_0^\infty \frac{x^\alpha}{x^2-1} dx = \frac{\pi[1-\cos(\pi\alpha)]}{2\sin(\pi\alpha)}.$$

Notice: You have to write down complete analysis to all parts of the integral (tell the reason even if the value of the part is zero).

10. <2016u4> Evaluate each of the following integrals:

(a) $\displaystyle\int_0^{2\pi} \frac{\cos 2\theta}{13 - 12\cos\theta} d\theta,$ (b) $\displaystyle\text{P.V.} \int_{-\infty}^\infty \frac{x\cos x}{x^2 - 3x + 2} dx.$

<2018u4>

1. <2018u4> Let $f(z) = 1/(\sqrt{2}\cos z - 1 + z - \frac{\pi}{4})^2$.

(a) Find the order of the pole of $f(z)$ at $z = \frac{\pi}{4}$.

(b) Evaluate $\displaystyle\oint_C \frac{f'(z)}{f(z)} dz$, where $C : |z - \frac{\pi}{4}| = 1$.

2. <2018u4> Evaluate each of the following contour integrals:

(a) $\displaystyle\oint_{|z|=1} \cot z dz,$ (b) $\displaystyle\oint_{|z|=1} \frac{\csc z}{z^2} dz,$

(c) $\displaystyle\oint_{|z|=4} \frac{z-1}{\sin z} dz,$ (d) $\displaystyle\oint_{|z|=2} \left(\frac{z-1}{z+1}\right)^3 dz.$

3. <2018u4> Find the results for ω in the upper or lower half-plane,

$$\text{P.V.} \int_{-\infty}^\infty \frac{\cos x}{x - \omega} dx.$$

4. <2018u4> Evaluate each of the following integrals:

(a) $\displaystyle\int_0^\pi \frac{d\theta}{(3+2\cos\theta)^2},$ (b) $\displaystyle\int_0^\infty \frac{x^2+1}{x^4+1} dx,$ (c) $\displaystyle\text{P.V.} \int_{-\infty}^\infty \frac{x}{(x^2+4x+13)^2} dx.$

5. <2018u4> Evaluate the integral $\displaystyle\oint_C \frac{f'(z)}{f(z)} dz$ for the following functions:

(a) $f(z) = \dfrac{(z-3iz-2)^2}{z(z^2-2z+2)^5}$, C is $|z| = \dfrac{3}{2}$,
(b) $f(z) = z^6 - 2iz^4 + (5-i)z^2 + 10$, C encloses all the zeros of f.

6. <2018u4> Evaluate

$$\text{P.V.} \int_0^\infty \frac{dx}{\sqrt{x}(x^2-4)}.$$

Notice: You have to write down complete analysis to all parts of the integral (tell the reason even if the value of the part is zero).

7. <2018u4> Evaluate each of the following integrals:

(a) $\text{P.V.} \displaystyle\int_{-\infty}^{\infty} \frac{x\sin(3x)}{x^4+4}dx$,
(b) $\text{P.V.} \displaystyle\int_0^{\infty} \frac{\sin(2x)}{x(x^2+1)^2}dx$,
(c) $\displaystyle\int_0^{2\pi} \frac{\cos 2\theta}{5-4\sin\theta}d\theta$,
(d) $\text{P.V.} \displaystyle\int_{-\infty}^{\infty} \frac{\sin(3x)}{x-2i}dx$.

<2019u4>

1. <2019u4> Suppose that $f(z)$ is analytic and has zero of order m at the point z_0. Show that the integral $\dfrac{1}{2\pi i}\displaystyle\oint_C \frac{f'(z)}{f(z)}dz = m$ for C enclosing z_0.

2. <2019u4> Compute the residue at each singularity of the following functions:

(a) $f(z) = \dfrac{\cos z}{z^2(z-\pi)^3}$,
(b) $\dfrac{z-1}{\sin z}$.

3. <2019u4> Evaluate $\displaystyle\oint_C e^{1/z}\sin\left(\frac{1}{z}\right)dz$, where C is |z|=1.

4. <2019u4> Evaluate each of the following integrals:

(a) $\text{P.V.} \displaystyle\int_{-\infty}^{\infty} \frac{x\sin x}{x^2-2x+10}dx$,
(b) $\text{P.V.} \displaystyle\int_{-\infty}^{\infty} \frac{\cos x}{(x^2+1)(x^2+9)}dx$,
(c) $\displaystyle\int_0^{2\pi} \frac{\sin^2\theta}{5+4\cos\theta}d\theta$,
(d) $\displaystyle\int_0^{\infty} \frac{dx}{\sqrt{x}(x+4)}$.

<2020u4>

1. <2020u4> Evaluate the integral $\displaystyle\int_0^{2\pi} \frac{1}{10 - 6\cos\theta} d\theta$.

2. <2020u4> Evaluate the integral P.V. $\displaystyle\int_0^{\infty} \frac{x\sin x}{(x^2-1)(x^2+4)} dx$.

3. <2020u4> Evaluate the integral P.V. $\displaystyle\int_0^{\infty} \frac{\sqrt{x}}{x^2+1} dx$.

4. <2020u4> Find the piecewise smooth function $f(t)$ with Laplace transform of $F(s) = \mathscr{L}\{f(t)\} = \dfrac{1}{s^4 + 2s^2 + 1}$.

5. <2020u4> Find the function $f(t)$ with Fourier transform of $F(\omega) = \mathscr{F}\{f(t)\} = \dfrac{\omega^2}{\pi(\omega^4 - 1)}$.