# Frequency Response

張大中
中央大學 通訊工程系
dcchang@ce.ncu.edu.tw

# Amplifier Frequency Response

- Amplifier Gain Versus Frequency

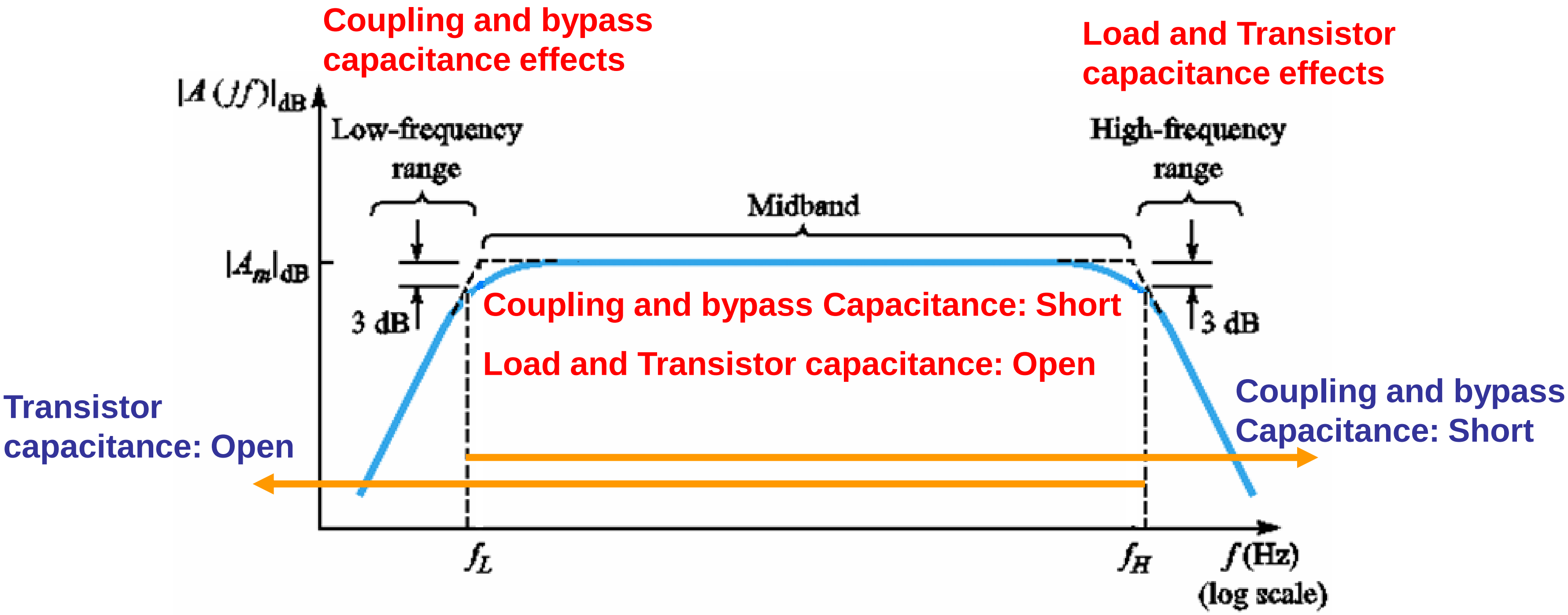



Figure 7.1 Amplifier gain versus frequency

- Audio Amplifiers and Speakers: Signal frequencies in the range of 20 Hz < f <20 kHz need to be amplified equally so as to reproduce the sound as accurately as possible. The frequency $f_L$ must be designed to be less than 20 Hz and $f_H$ must be designed to be greater than 20 kHz.

# S-Domain Analysis

- Transfer Function

$$T(s) = K\frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)}$$

- Zeros

$$z_1, z_2, \cdots, z_m$$

Each capacitor is represented by its complex impedance 1/sC, and each inductor by sL.

- Poles

$$p_1, p_2, \cdots, p_m$$

- Frequency

$$s = j\omega = j2\pi f$$

- Magnitude and Phase of the Transfer Function

$$T(j\omega) = |T(j\omega)|\, e^{-j\angle\Phi(j\omega)}$$

$$|T(j\omega)|: \text{magnitude}, \angle\Phi(j\omega): \text{Phase}$$

# Time Constant for a Simple Transfer Function

- Time Constant

$$T(s) = K_1\left(\frac{1}{1+s\tau_1}\right) \xrightarrow{\mathcal{L}^{-1}} \frac{K_1}{\tau_1}e^{-t/\tau_1}$$

$\tau_1$ is a time constant

$$T(s) = K_2\left(\frac{s\tau_2}{1+s\tau_2}\right) \xrightarrow{\mathcal{L}^{-1}} K_2\left[\delta(t) - \frac{1}{\tau_2}e^{-t/\tau_2}\right]$$

$\tau_2$ is a time constant

# Series Coupling Capacitor Circuits

- Series Coupling Capacitor Circuit

$$\frac{V_o(s)}{V_i(s)} = \frac{R_P}{R_S + R_P + 1/sC_S}$$

$$= \frac{sR_P C_S}{1 + s(R_S + R_P)C_S}$$

$$= \frac{R_P}{R_S + R_P} \cdot \frac{s(R_S + R_P)C_S}{1 + s(R_S + R_P)C_S}$$

$$= K\frac{s\tau}{1 + s\tau}$$

Time Constant: $\tau = (R_S + R_P)C_S$

High-pass Network

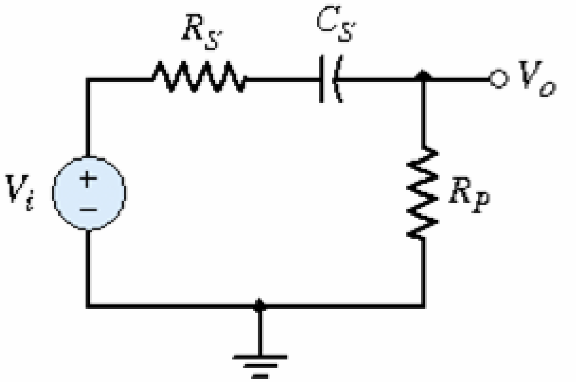



Figure 7.2 Series coupling capacitor circuit

# Capacitor Circuits

- Parallel Load Capacitor Circuit

$$\frac{V_o(s)}{V_i(s)} = \frac{R_P//(1/sC_P)}{R_S + (R_P//(1/sC_P))}$$

$$= \frac{1}{1 + R_S \Big/ \left( \dfrac{1}{R_P//(1/sC_P)} \right)}$$

$$= \frac{1}{1 + R_S(1/R_P + sC_P)} = \frac{1}{\dfrac{R_P + R_S}{R_P} + sC_P R_P}$$

$$= \frac{R_P}{R_P + R_S} \cdot \frac{1}{1 + s(R_S//R_P)C_P}$$

$$= K\frac{1}{1 + s\tau}$$

Time Constant: $\tau = (R_S // R_P)C_P$

Low-pass Network

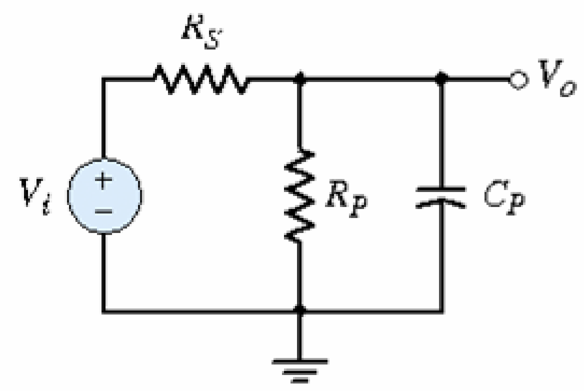



**Figure 7.3** Parallel load capacitor circuit

# Bode Plots for the Series Coupling Capacitor Circuit

- Amplitude Frequency Response

$$T(j\omega)=\left(\frac{R_P}{R_S+R_P}\right)\left[\frac{j\omega\tau_S}{1+j\omega\tau_S}\right]$$

$$|T(jf)|=\left(\frac{R_P}{R_S+R_P}\right)\left[\frac{2\pi f\tau_S}{\sqrt{1+(2\pi f\tau_S)^2}}\right]$$

$$|T(jf)|_{dB}=20\log_{10}\left[\left(\frac{R_P}{R_S+R_P}\right)\cdot\frac{2\pi f\tau_S}{\sqrt{1+(2\pi f\tau_S)^2}}\right]$$

$$=20\log_{10}\left(\frac{R_P}{R_S+R_P}\right)+20\log_{10}(2\pi f\tau_S)$$

$$-20\log_{10}\sqrt{1+(2\pi f\tau_S)^2}$$

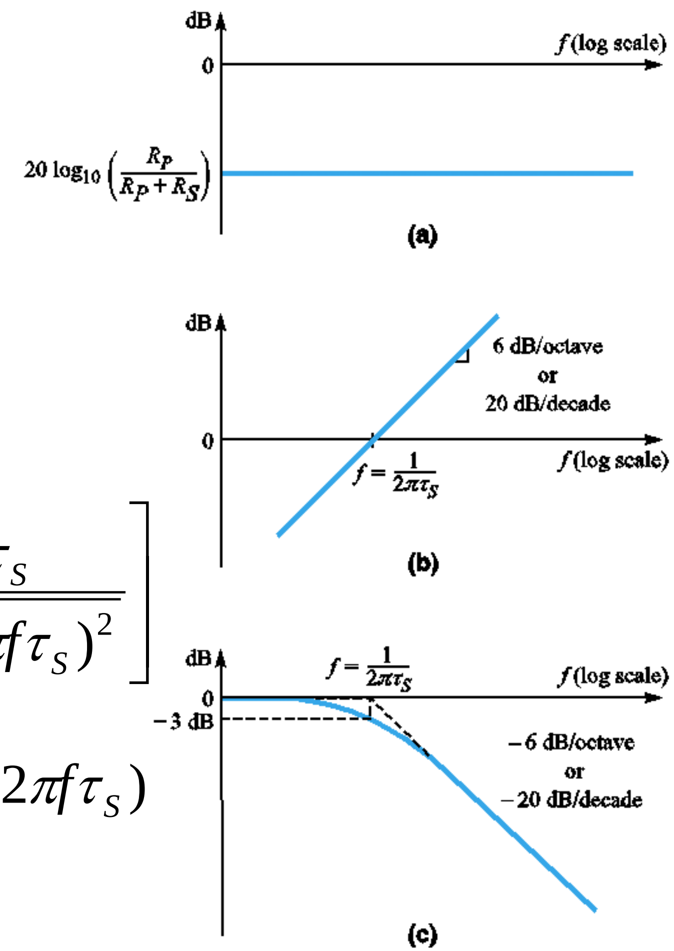


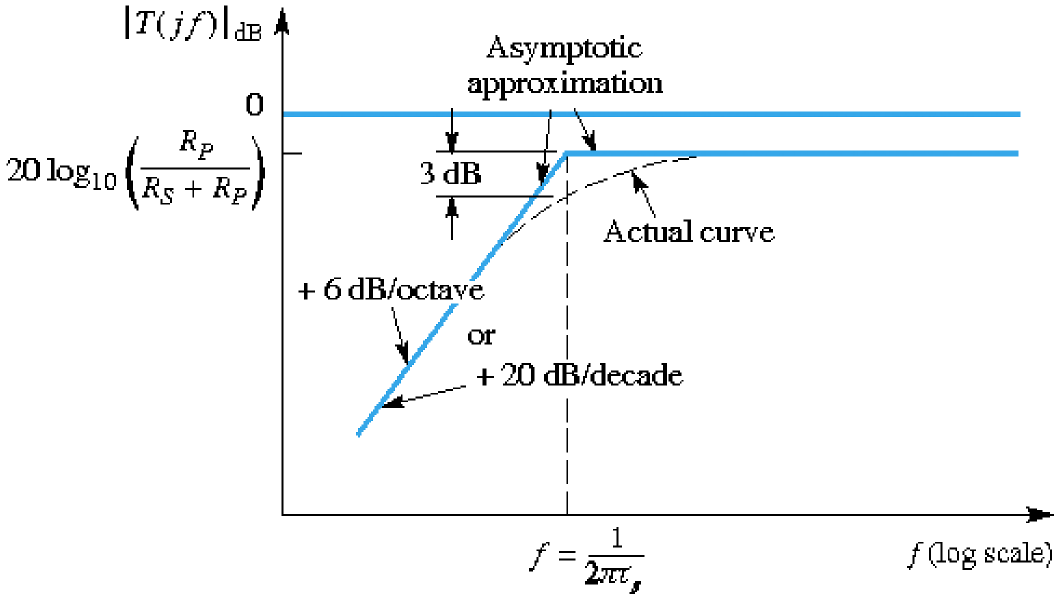

**Figure 7.4** Plots of (a) the first term, (b) the second term, and (c) the third term of Equation (7.10(b))

# Bode Plots for the Series Coupling Capacitor Circuit

- Amplitude Frequency Response
  - ✓ Break-point frequency, corner frequency, -3dB frequency: $f = 1/2\pi\tau_S$

$f << 1/2\pi\tau_S$ The capacitor approaches an open circuit, and the output voltage approaches zero.

$f >> 1/2\pi\tau_S$ The capacitor approaches a short circuit, and the output voltage gain is $R_P/(R_S + R_P)$.




High-pass Network

**Figure 7.5** Bode plot of the voltage transfer function magnitude for the circuit in Figure 7.2

# Bode Plots for the Series Coupling Capacitor Circuit

- Phase Frequency Response

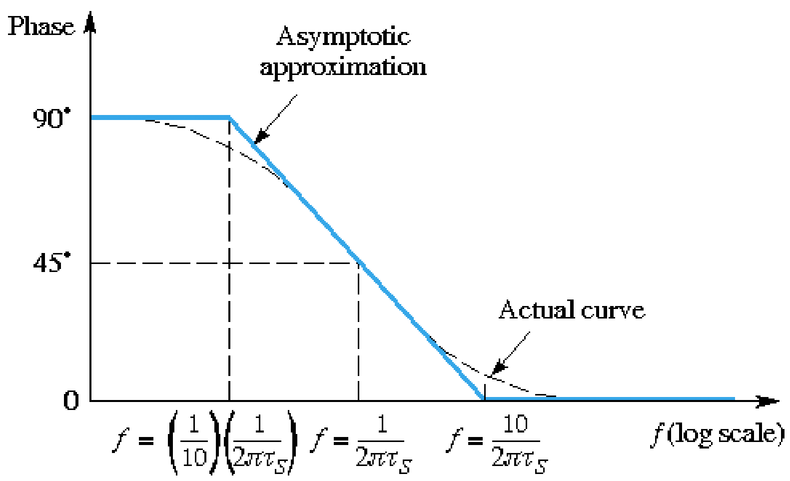
$$A + jB = Ke^{j\theta}, K = \sqrt{A^2 + B^2}, \theta = \tan^{-1}(B/A)$$

$$T(jf) = \left(\frac{R_P}{R_S + R_P}\right)\left[\frac{j2\pi f\tau_S}{1 + j2\pi f\tau_S}\right] = \left[\left|\frac{R_P}{R_S + R_P}\right| e^{j\theta_1}\right] \frac{|j2\pi f\tau_S| e^{j\theta_2}}{|1 + j2\pi f\tau_S| e^{j\theta_3}}$$

$$= \frac{K_1 K_2}{K_3} e^{j(\theta_1 + \theta_2 - \theta_3)}$$

$$\theta = \theta_1 + \theta_2 - \theta_3$$

$$= 90^o - \tan^{-1}(2\pi f\tau_S)$$




$$\tan^{-1}(0) = 0$$

$$\tan^{-1}(1) = 45^o$$

$$\tan^{-1}(\infty) = 90^o$$

**Figure 7.7** Bode plot of the voltage transfer function phase for the circuit in Figure 7.2

# Bode Plots for the Parallel Load Capacitor Circuit

- Amplitude Frequency Response

$$T(jf) = \left(\frac{R_P}{R_S + R_P}\right)\left[\frac{1}{1 + j2\pi f \tau_P}\right]$$

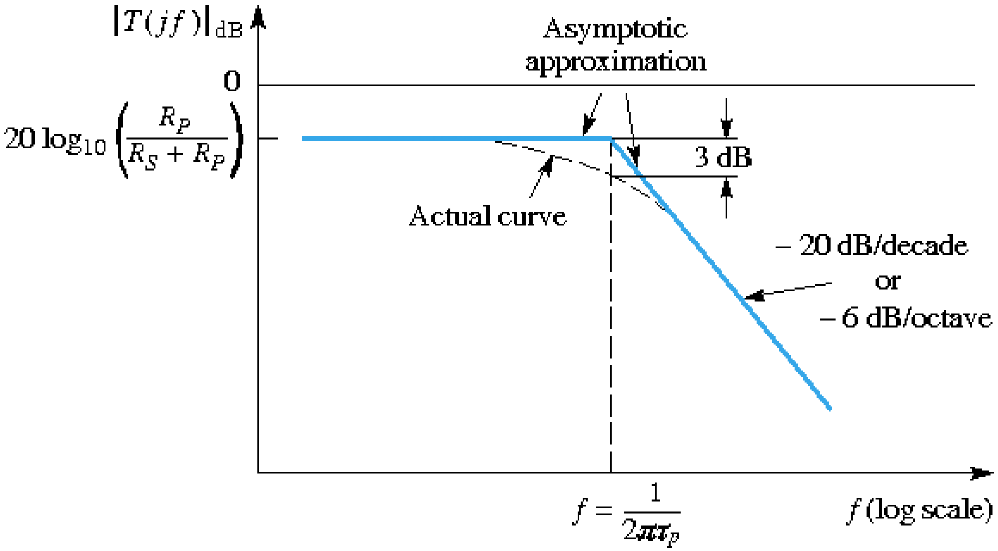
$f \ll 1/2\pi\tau_S$

The capacitor approaches a short circuit, and the output voltage gain is $R_P/(R_S + R_P)$ .

$$|T(jf)| = \left(\frac{R_P}{R_S + R_P}\right)\left[\frac{1}{\sqrt{1 + (2\pi f \tau_P)^2}}\right]$$

$f \gg 1/2\pi\tau_S$

The capacitor approaches an open circuit, and the output voltage approaches zero.




Low-pass Network

**Figure 7.8** Bode plot of the voltage transfer function magnitude for the circuit in Figure 7.3

# Bode Plots for the Parallel Load Capacitor Circuit

- Phase Frequency Response

$$\text{Phase} = -\angle \tan^{-1}(2\pi f \tau_P)$$

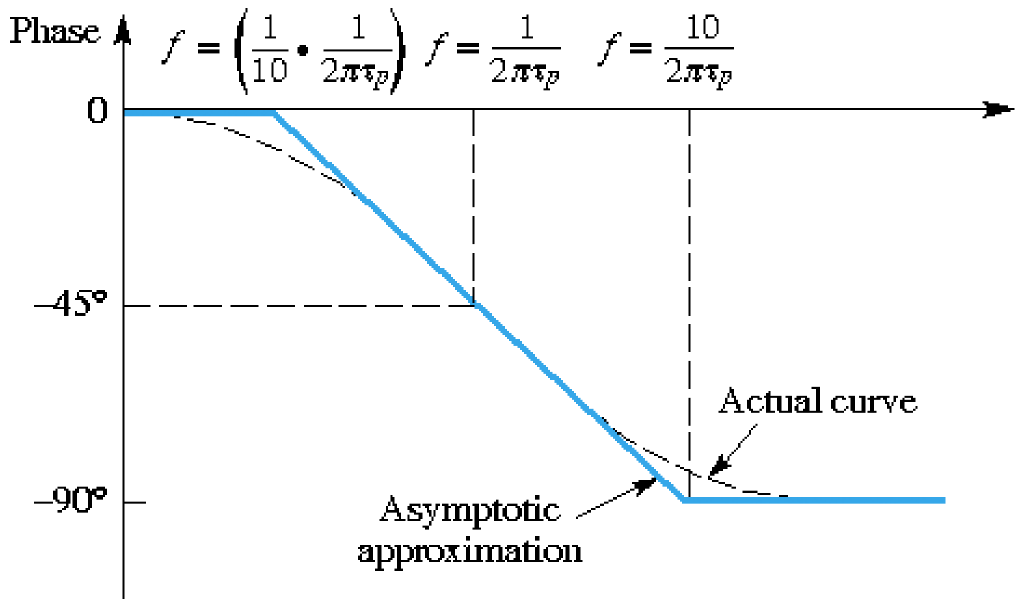



Figure 7.9 Bode plot of the voltage transfer function phase for the circuit in Figure 7.3

**Example 7.1** **Objective:** Determine the corner frequencies and maximum-magnitude asymptotes of the Bode plots for a specified circuit.

For the circuits in Figures 7.2 and 7.3, the parameters are: $R_S = 1\,\text{k}\Omega$, $R_P = 10\,\text{k}\Omega$, $C_S = 1\,\mu\text{F}$, and $C_P = 3\,\text{pF}$.

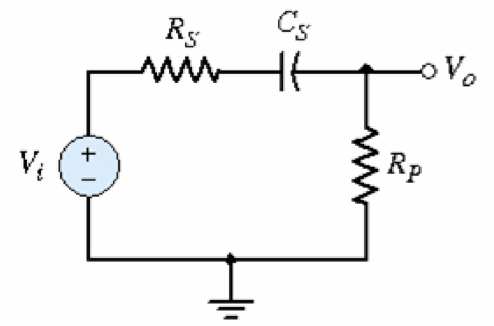



Figure 7.2 Series coupling capacitor circuit

**Solution:** (Figure 7.2) The time constant is

$$\tau_S = (R_S + R_P)C_S = (10^3 + 10 \times 10^3)(10^{-6}) = 1.1 \times 10^{-2}\,\text{s}$$

or

$$\tau_S = 11\,\text{ms}$$

The corner frequency of the Bode plot shown in Figure 7.5 is then

$$f = \frac{1}{2\pi\tau_S} = \frac{1}{(2\pi)(11 \times 10^{-3})} = 14.5\,\text{Hz}$$

The maximum magnitude is

$$\frac{R_P}{R_S + R_P} = \frac{10}{1 + 10} = 0.909$$

or

$$20\log_{10}\left(\frac{R_P}{R_S + R_P}\right) = -0.828\,\text{dB}$$

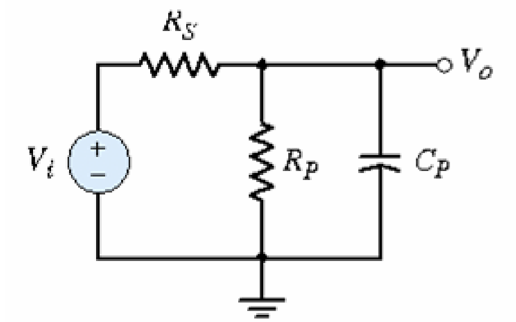



Figure 7.3 Parallel load capacitor circuit

**Solution:** (Figure 7.3) The time constant is

$$\tau_P = (R_S \| R_P)C_P = (10^3 \| (10 \times 10^3))(3 \times 10^{-12}) = 2.73 \times 10^{-9}\,\text{s}$$

or

$$\tau_P = 2.73\,\text{ns}$$

The corner frequency of the Bode plot in Figure 7.8 is then

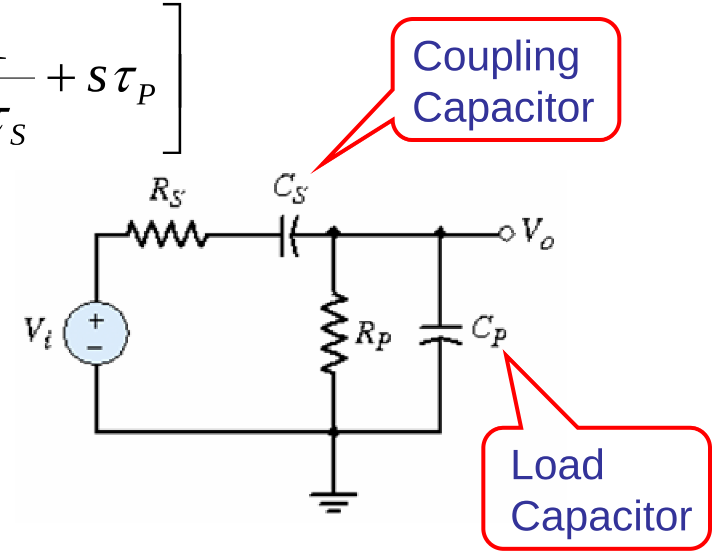
$$f = \frac{1}{2\pi\tau_P} = \frac{1}{(2\pi)(2.73 \times 10^{-9})} \Rightarrow 58.3\,\text{MHz}$$

The maximum magnitude is the same as just calculated: 0.909 or −0.828 dB.

# Short-Circuit and Open-Circuit Time Constants

- Circuit with both a series coupling and a parallel load capacitor

$$\frac{V_o(s)}{V_i(s)} = \left(\frac{R_P}{R_S + R_P}\right) \times \frac{1}{\left[1 + \left(\frac{R_P}{R_S + R_P}\right)\left(\frac{C_P}{C_S}\right) + \frac{1}{s\tau_S} + s\tau_P\right]}$$




At Low Frequency: $C_P$ is treated as an open circuit

$$\tau_S = (R_S + R_P)C_S$$

Open-circuit time constant

$$f_L = \frac{1}{2\pi\tau_S}$$

At High Frequency: $C_S$ is treated as a short circuit

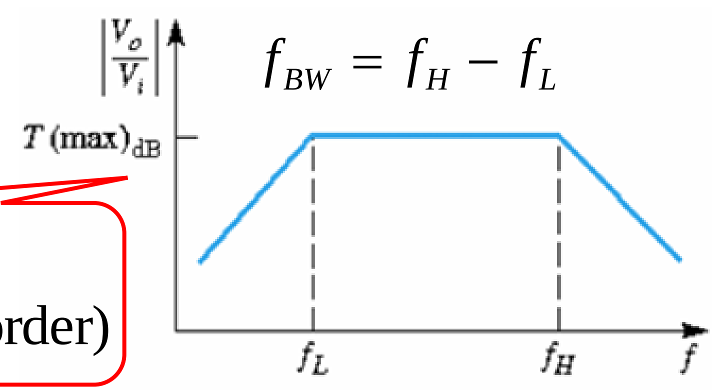
$$\tau_P = (R_S // R_P)C_P$$

Short-circuit time constant

$$f_H = \frac{1}{2\pi\tau_P}$$

$$f_H > f_L \Rightarrow \tau_P < \tau_S$$

$$C_S > C_P \quad (R_S, R_P \text{ the same order})$$



**Example 7.2** **Objective:** Determine the corner frequencies and bandwidth of a passive circuit containing two capacitors.

Consider the circuit shown in Figure 7.10 with parameters $R_S = 1\,\text{k}\Omega$, $R_P = 10\,\text{k}\Omega$, $C_S = 1\,\mu\text{F}$, and $C_P = 3\,\text{pF}$.

**Solution:** Since $C_P$ is less than $C_S$ by approximately six orders of magnitude, we can treat the effect of each capacitor separately. The open-circuit time constant is

$$\tau_S = (R_S + R_P)C_S = (10^3 + 10 \times 10^3)(10^{-6}) = 1.1 \times 10^{-2}\ \text{s}$$

and the short-circuit time constant is

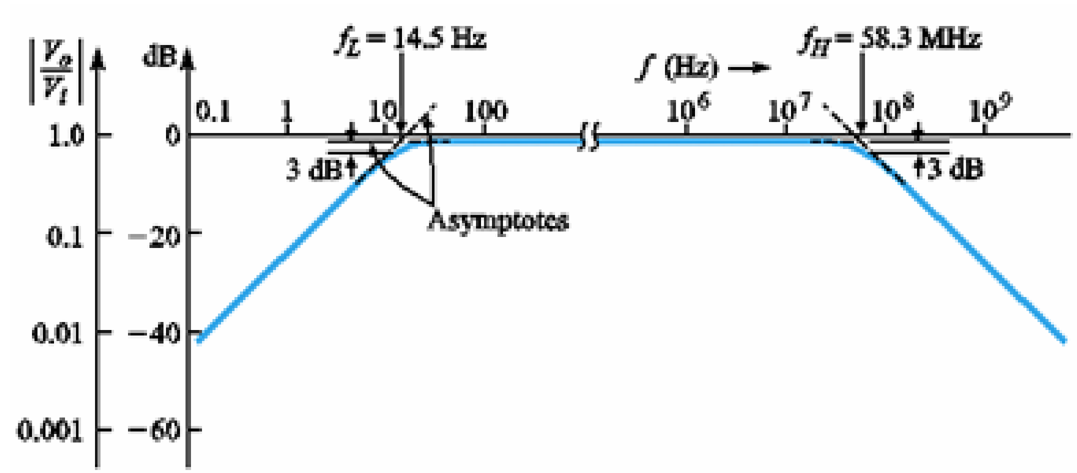
$$\tau_P = (R_S \| R_P)C_P = [10^3 \| (10 \times 10^3)](3 \times 10^{-12}) = 2.73 \times 10^{-9}\ \text{s}$$

The corner frequencies are then

$$f_L = \frac{1}{2\pi\tau_S} = \frac{1}{2\pi(1.1 \times 10^{-2})} = 14.5\ \text{Hz}$$

and

$$f_H = \frac{1}{2\pi\tau_P} = \frac{1}{2\pi(2.73 \times 10^{-9})} \Rightarrow 58.3\ \text{MHz}$$




Finally, the bandwidth is

$$f_{BW} = f_H - f_L = 58.3\ \text{MHz} - 14.5\ \text{Hz} \cong 58.3\ \text{MHz}$$

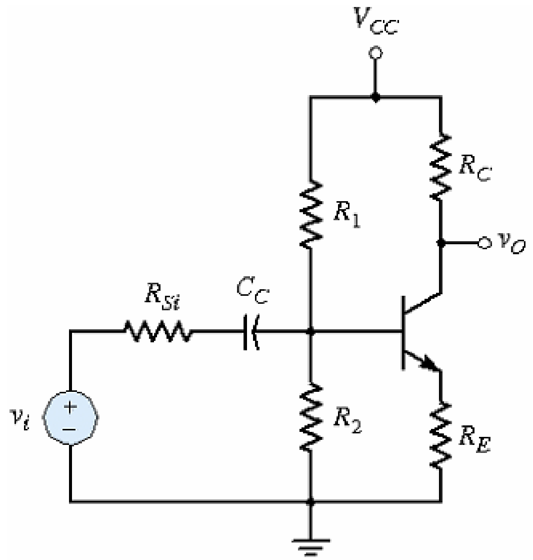
**Comment:** The corner frequencies in this example are exactly the same as those determined in Example 7.1. This occurred because the two corner frequencies are far apart. The maximum magnitude of the voltage transfer function is again

$$\frac{R_P}{R_S + R_P} = \frac{10}{1 + 10} = 0.909 \Rightarrow -0.828\ \text{dB}$$

# Coupling Capacitor Effects

- Input Coupling Capacitor: Common-Emitter Circuit

$$V_\pi = \frac{R_i V_i}{R_i + R_{Si} + \frac{1}{sC_C}} \times \frac{r_\pi}{R_{ib}}$$

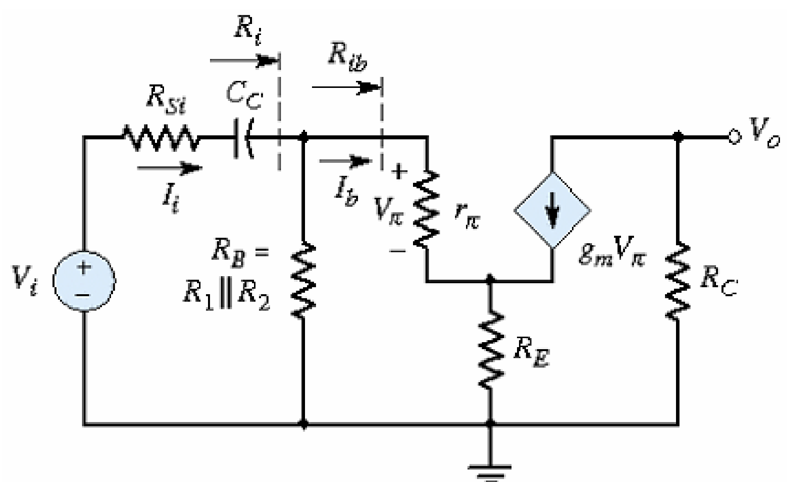



$$A_v(s) = \frac{V_o(s)}{V_i(s)} = -g_m V_\pi R_C / V_i$$

$$= -\frac{R_i}{R_i + R_{Si} + \frac{1}{sC_C}} \cdot \frac{r_\pi}{R_{ib}} \cdot g_m R_C$$

$$= -\frac{R_i r_\pi g_m R_C}{R_{ib}} \cdot \frac{sC_C}{1 + s(R_i + R_{Si})C_C}$$

$$= -\frac{R_i r_\pi g_m R_C}{R_{ib}(R_i + R_{Si})} \cdot \frac{s\tau_S}{1 + s\tau_S}$$




$$\tau_S = (R_i + R_{Si})C_C \qquad \text{Note}: \frac{R_i}{R_{ib}} = \frac{R_B // R_{ib}}{R_{ib}} = \frac{R_B}{R_B + R_{ib}} \qquad f_L = \frac{1}{2\pi(R_{Si} + R_i)C_C}$$

**Example 7.3** **Objective:** Calculate the corner frequency and maximum gain of a bipolar common-emitter circuit with a coupling capacitor.

For the circuit shown in Figure 7.16, the parameters are: $R_1 = 51.2\,\text{k}\Omega$, $R_2 = 9.6\,\text{k}\Omega$, $R_C = 2\,\text{k}\Omega$, $R_E = 0.4\,\text{k}\Omega$, $R_{Si} = 0.1\,\text{k}\Omega$, $C_C = 1\,\mu\text{F}$, and $V_{CC} = 10\,\text{V}$. The transistor parameters are: $V_{BE}(\text{on}) = 0.7\,\text{V}$, $\beta = 100$, and $V_A = \infty$.

**Solution:** From a dc analysis, the quiescent collector current is $I_{CQ} = 1.81\,\text{mA}$. The transconductance is therefore

$$g_m = \frac{I_{CQ}}{V_T} = \frac{1.81}{0.026} = 69.6\,\text{mA/V}$$

and the diffusion resistance is

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{1.81} = 1.44\,\text{k}\Omega$$

The input resistance is

$$R_i = R_1 \| R_2 \| [r_\pi + (1+\beta)R_E]$$
$$= 51.2 \| 9.6 \| [1.44 + (101)(0.4)] = 6.77\,\text{k}\Omega$$

and the time constant is therefore

$$\tau_S = (R_{Si} + R_i)C_C = (0.1 \times 10^3 + 6.77 \times 10^3)(1 \times 10^{-6}) = 6.87 \times 10^{-3}\,\text{s}$$

or

$$\tau_S = 6.87\,\text{ms}$$

The corner frequency is

$$f_L = \frac{1}{2\pi\tau_S} = \frac{1}{2\pi(6.87 \times 10^{-3})} = 23.2\,\text{Hz}$$

Finally, the maximum voltage gain magnitude is

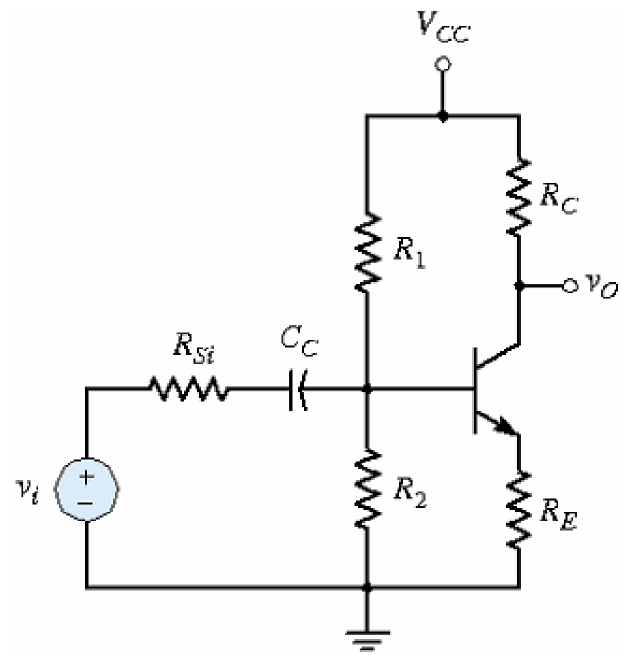
$$|A_v|_{\max} = \frac{g_m r_\pi R_C}{(R_{Si} + R_i)}\left(\frac{R_B}{R_B + R_{ib}}\right)$$

where

$$R_{ib} = r_\pi + (1+\beta)R_E = 1.44 + (101)(0.4) = 41.8\,\text{k}\Omega$$

Therefore,

$$|A_v|_{\max} = \frac{(69.6)(1.44)}{(0.1 + 6.77)}\left(\frac{8.08}{8.08 + 41.8}\right) = 4.73$$



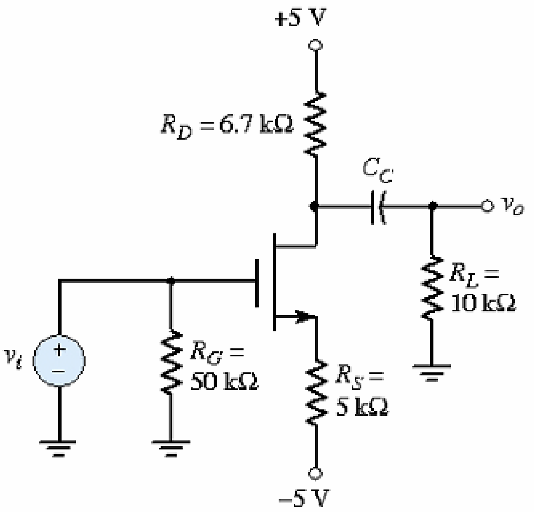
# Coupling Capacitor Effects

- Output Coupling Capacitor: Common-Source Circuit
  - ✓ Maximum Small-Signal Gain ( Assume that $C_C$ is short)

$$|V_o|_{\max} = g_m V_{gs}(R_D // R_L)$$

$$V_i = V_{gs} + g_m V_{gs} R_S = (1 + g_m R_S) V_{gs}$$

$$|A_v|_{\max} = \frac{g_m (R_D // R_L)}{1 + g_m R_S}$$

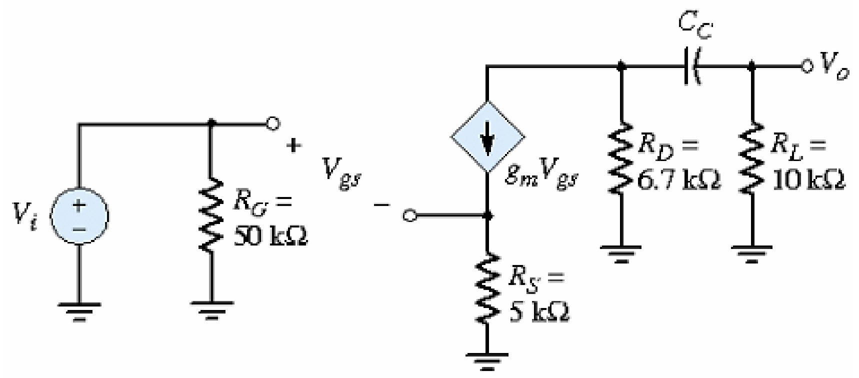



  - ✓ Time Constant (High-pass Network)

$$\text{set } V_i = 0, \text{ then } V_{gs} = 0$$

$$\tau_S = (R_D + R_L) C_C$$

$$f_L = 1 / 2\pi\tau_S$$



**Design Example 7.4** **Objective:** The circuit in Figure 7.17(a) is to be used as a simple audio amplifier. Design the circuit such that the lower corner frequency is $f_L = 20\,\text{Hz}$.

**Solution:** The corner frequency can be written in terms of the time constant, as follows:

$$f_L = \frac{1}{2\pi\tau_S}$$

The time constant is then

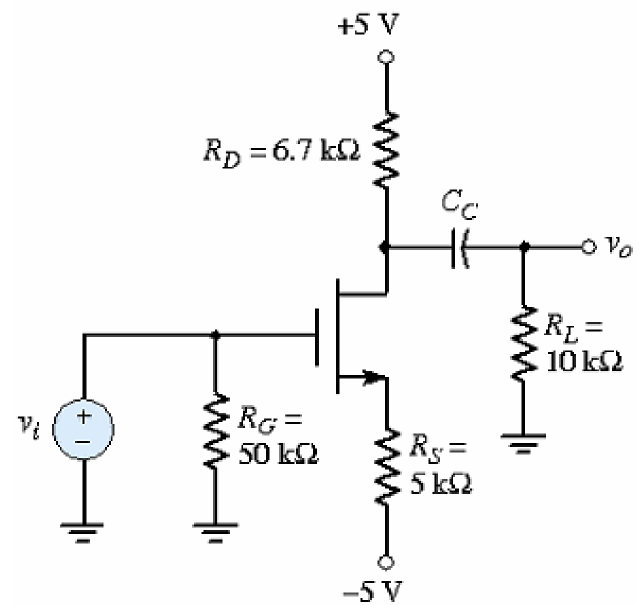
$$\tau_S = \frac{1}{2\pi f} = \frac{1}{2\pi(20)} \Rightarrow 7.96\,\text{ms}$$

Therefore, from Equation (7.37) the coupling capacitance is

$$C_C = \frac{\tau_S}{R_D + R_L} = \frac{7.96 \times 10^{-3}}{6.7 \times 10^3 + 10 \times 10^3} = 4.77 \times 10^{-7}\,\text{F}$$

or

$$C_C = 0.477\,\mu\text{F}$$




**Comment:** Using the time constant technique to find the corner frequency is substantially easier than using the circuit analysis approach.

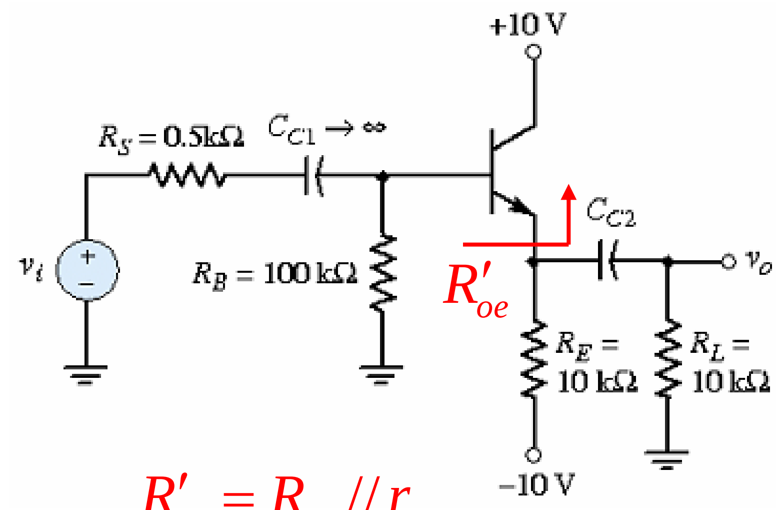
# Coupling Capacitor Effects

- Output Coupling Capacitor: Emitter-Follower Circuit

$$\frac{V_x}{r_\pi + R_B // R_S} = g_m V_\pi + I_x$$

$$= -\frac{g_m r_\pi}{r_\pi + R_B // R_S} V_x + I_x$$

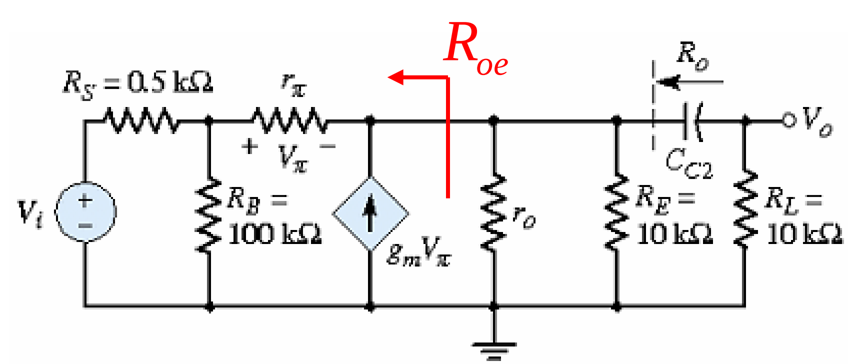
$$\frac{1+\beta}{r_\pi + R_B // R_S} V_x = I_x$$




$$R'_{oe} = R_{oe} // r_o$$

$$R_{oe} = \frac{V_x}{I_x} = \frac{r_\pi + R_B // R_S}{1+\beta}$$

$$R_o = R_E // r_o // \left\{ \frac{r_\pi + R_B // R_S}{1+\beta} \right\}$$




Time Constant: $\tau_S = [R_o + R_L] C_{C2}$

**Example 7.5** **Objective:** Determine the 3 dB frequency of an emitter-follower amplifier circuit with an output coupling capacitor.

Consider the circuit shown in Figure 7.18(a) with transistor parameters $\beta = 100$, $V_{BE}(\text{on}) = 0.7\ \text{V}$, and $V_A = 120\ \text{V}$. The output coupling capacitance is $C_{C2} = 1\ \mu\text{F}$.

**Solution:** A dc analysis shows that $I_{CQ} = 0.838\ \text{mA}$. Therefore, the small-signal parameters are: $r_\pi = 3.10\ \text{k}\Omega$, $g_m = 32.2\ \text{mA/V}$, and $r_o = 143\ \text{k}\Omega$.

From Equation (7.39), the output resistance $R_o$ of the emitter follower is

$$R_o = R_E \| r_o \| \left\{ \frac{[r_\pi + (R_S \| R_B)]}{1+\beta} \right\}$$

$$= 10 \| 143 \| \left\{ \frac{[3.10 + (0.5 \| 100)]}{101} \right\} = 10 \| 143 \| 0.0356\ \text{k}\Omega$$

or

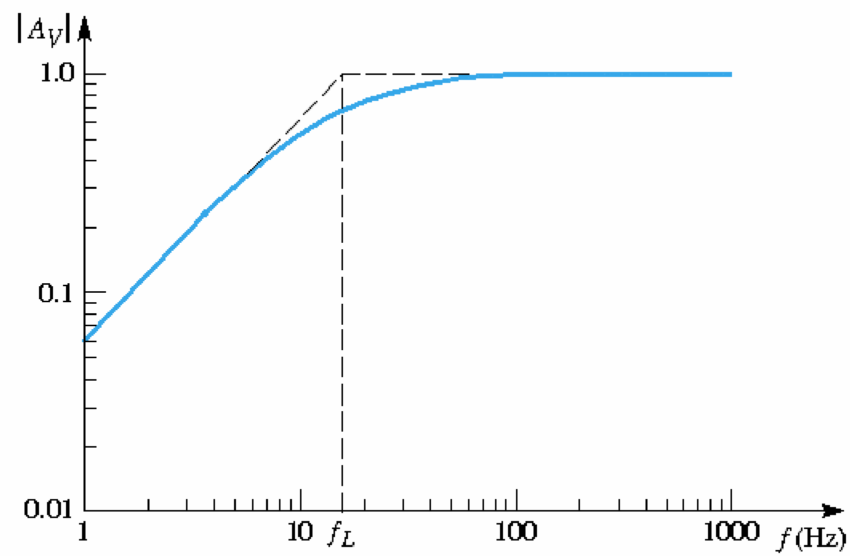
$$R_o \cong 35.5\ \Omega$$

From Equation (7.38), the time constant is

$$\tau_S = [R_o + R_L]C_{C2} = [35.5 + 10^4](10^{-6}) \cong 1 \times 10^{-2}\ \text{s}$$

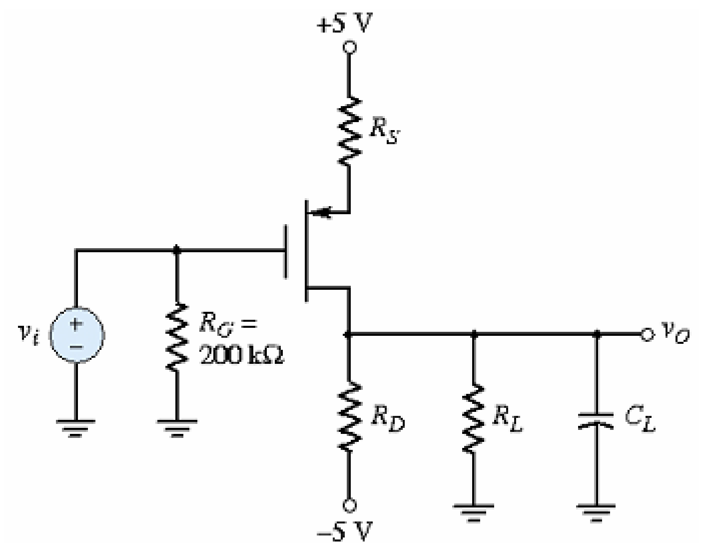
The 3 dB frequency is then

$$f_L = \frac{1}{2\pi\tau_S} = \frac{1}{2\pi(10^{-2})} = 15.9\ \text{Hz}$$



# Load Capacitor Effects

- The model of the load circuit input impedance is generally a capacitance in parallel with a resistance.
- There is a parasitic capacitance between ground and the line that connects the amplifier output to the load circuit.




$$f \to \infty,\ C_L : \text{short},\ V_o = 0 \quad \text{(Lowpass)}$$

$$f \to 0,\ C_L : \text{open},\ A_v : \text{const.}$$

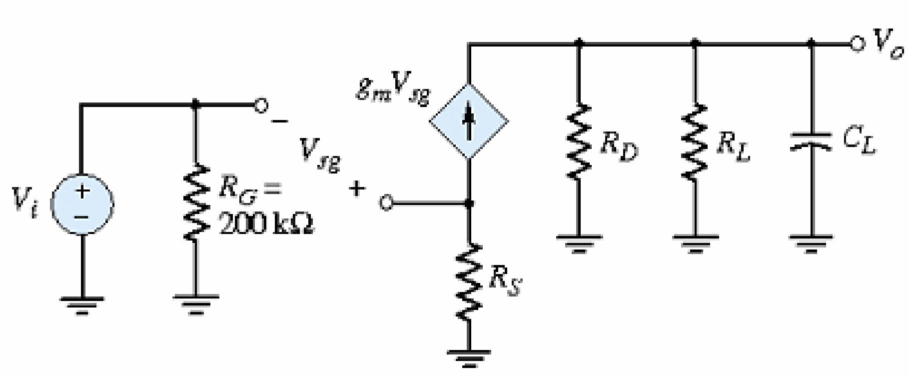
$$V_i = -V_{sg} - g_m R_S V_{sg}$$

$$= -(1 + g_m R_S) V_{sg}$$

$$V_o = g_m (R_D // R_L) V_{sg}$$

$$|A_v|_{\max} = \frac{g_m (R_D // R_L)}{1 + g_m R_S}$$

Time Constant: $\tau_P = (R_D // R_L) C_L$



**Example 7.6** **Objective:** Determine the corner frequency and maximum gain asymptote of a MOSFET amplifier.

For the circuit in Figure 7.20(a), the parameters are: $R_S = 3.2\,\text{k}\Omega$, $R_D = 10\,\text{k}\Omega$, $R_L = 20\,\text{k}\Omega$, and $C_L = 10\,\text{pF}$. The transistor parameters are: $V_{TP} = -2\,\text{V}$, $K_p = 0.25\,\text{mA/V}^2$, and $\lambda = 0$.

**Solution:** From the dc analysis, we find that $I_{DQ} = 0.5\,\text{mA}$, $V_{SGQ} = 3.41\,\text{V}$, and $V_{SDQ} = 3.41\,\text{V}$. The transconductance is therefore

$$g_m = 2K_p(V_{SG} + V_{TP}) = 2(0.25)(3.41 - 2) = 0.705\,\text{mA/V}$$

From Equation (7.40), the time constant is

$$\tau_P = (R_D \| R_L)C_L = ((10 \times 10^3)\|(20 \times 10^3))(10 \times 10^{-12}) = 6.67 \times 10^{-8}\,\text{s}$$

or

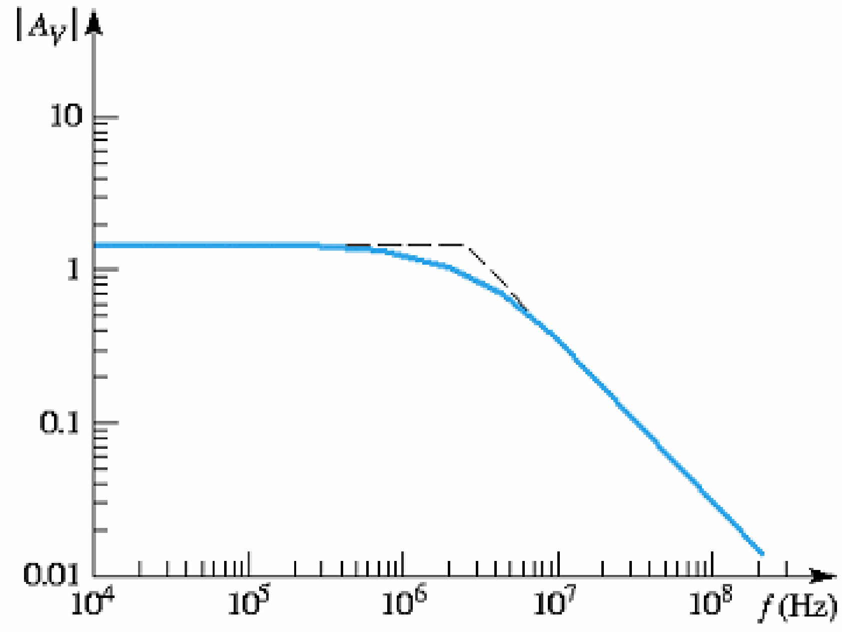
$$\tau_P = 66.7\,\text{ns}$$

Therefore, the corner frequency is

$$f_H = \frac{1}{2\pi\tau_P} = \frac{1}{2\pi(66.7 \times 10^{-9})} \Rightarrow 2.39\,\text{MHz}$$

Finally, from Equation (7.41), the maximum gain asymptote is

$$|A_v|_{\max} = \frac{g_m(R_D \| R_L)}{1 + g_m R_S} = \frac{(0.705)(10\|20)}{1 + (0.705)(3.2)} = 1.44$$



# Coupling and Load Capacitors

- The values of the coupling capacitance and load capacitance differ by orders of magnitude ($C_C >> C_L$).
- The lower corner frequency $f_L$ is dominated by $C_C$ and is given by $f_L = 1/2\pi\tau_S$.
- The upper corner frequency $f_H$ is dominated by $C_L$ and is given by $f_H = 1/2\pi\tau_P$.

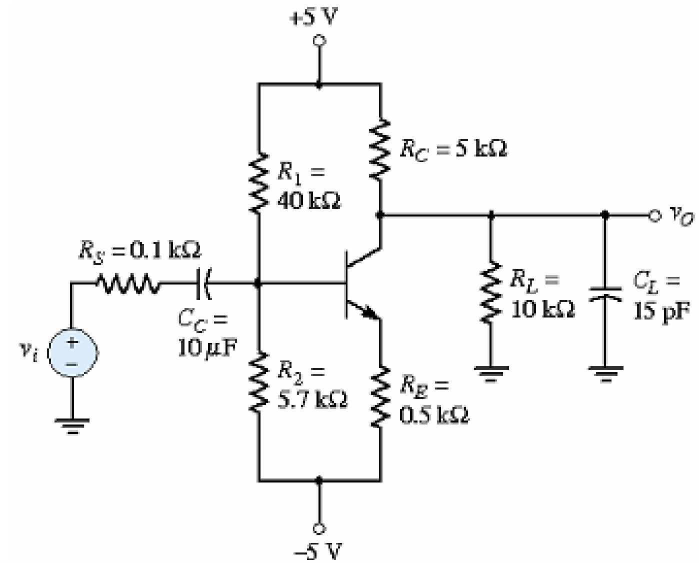
$$\tau_S = [R_S + R_1 // R_2 // R_i]C_C$$

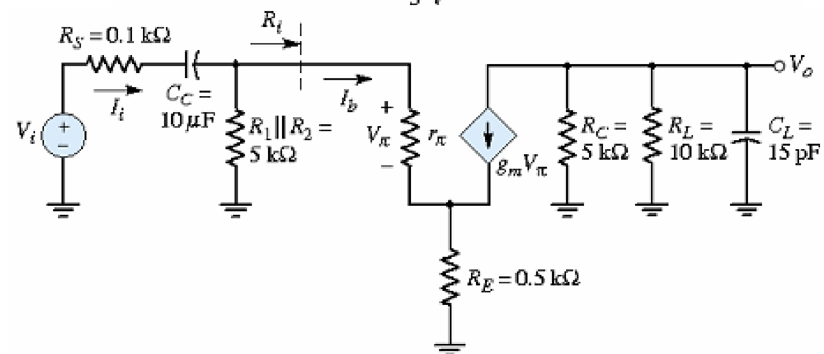
$$R_i = r_\pi + (1+\beta)R_E$$

$$\tau_P = (R_C // R_L)C_L$$

Midband: $C_C$ : short, $C_L$ : open

$$A_v = -\frac{R_1 // R_2 // R_i}{R_S + R_1 // R_2 // R_i} \cdot \frac{g_m r_\pi (R_C // R_L)}{R_i}$$



**Example 7.7** **Objective:** Determine the midband gain, corner frequencies, and bandwidth of a circuit containing both a coupling capacitor and a load capacitor.

Consider the circuit shown in Figure 7.22(a) with transistor parameters $V_{BE}(\text{on}) = 0.7\,\text{V}$, $\beta = 100$, and $V_A = \infty$.

**Solution:** The dc analysis of this circuit yields a quiescent collector current of $I_{CQ} = 0.99\,\text{mA}$. The transconductance is

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.99}{0.026} = 38.1\,\text{mA/V}$$

and the base diffusion resistance is

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.99} = 2.63\,\text{k}\Omega$$

The input resistance $R_i$ is therefore

$$R_i = r_\pi + (1+\beta)R_E = 2.63 + (101)(0.5) = 53.1\,\text{k}\Omega$$

From Equation (7.51), the midband gain is

$$|A_v|_{\max} = \left|\frac{V_o}{V_i}\right|_{\max} = g_m r_\pi (R_C \| R_L)\left(\frac{R_1 \| R_2}{(R_1 \| R_2) + R_i}\right)\left(\frac{1}{[R_S + (R_1 \| R_2 \| R_i)]}\right)$$

$$= (38.1)(2.63)(5\|10)\left(\frac{40\|5.7}{(40\|5.7) + 53.1}\right)\left(\frac{1}{[0.1 + (40\|5.7\|53.1)]}\right)$$

or

$$|A_v|_{\max} = 6.16$$

The time constant $\tau_S$ is

$$\tau_S = (R_S + R_1 || R_2 \| R_i)C_C$$
$$= (0.1 \times 10^3 + (5.7 \times 10^3)\|(40 \times 10^3)\|(53.1 \times 10^3))(10 \times 10^{-6}) = 4.66 \times 10^{-2}\,\text{s}$$

or

$$\tau_S = 46.6\,\text{ms}$$

and the time constant $\tau_P$ is

$$\tau_P = (R_C \| R_L)C_L = ((5 \times 10^3)\|(10 \times 10^3))(15 \times 10^{-12}) = 5 \times 10^{-8}\,\text{s}$$

or

$$\tau_P = 50\,\text{ns}$$

The lower corner frequency is

$$f_L = \frac{1}{2\pi\tau_S} = \frac{1}{2\pi(46.6 \times 10^{-3})} = 3.42\,\text{Hz}$$

and the upper corner frequency is

$$f_H = \frac{1}{2\pi\tau_P} = \frac{1}{2\pi(50 \times 10^{-9})} \Rightarrow 3.18\,\text{MHz}$$

Finally, the bandwidth is

$$f_{\text{BW}} = f_H - f_L = 3.18\,\text{MHz} - 3.4\,\text{Hz} \cong 3.18\,\text{MHz}$$

# Bypass Capacitor Effects

- The emitter bypass capacitor is often included so that the emitter resistor can be used to stabilize the Q-point without sacrificing the small-signal gain.

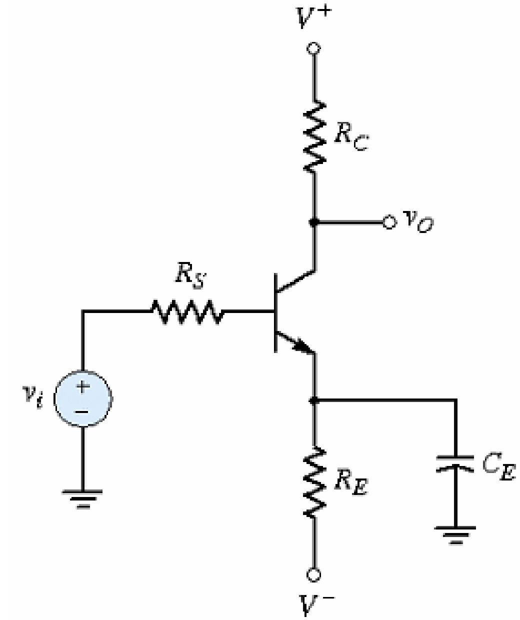
$$A_v(s) = \frac{-\beta R_C}{R_S + r_\pi + (1+\beta)\left(R_E \,//\, \dfrac{1}{sC_E}\right)}$$

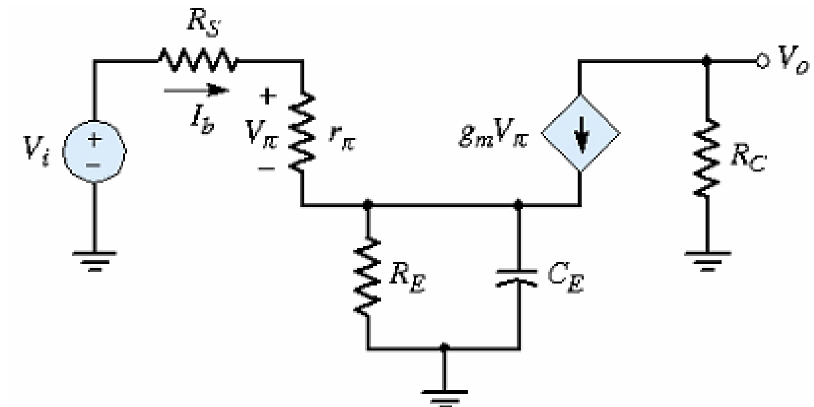
$$= \frac{-\beta R_C}{R_S + r_\pi + (1+\beta)R_E} \times \frac{1 + sR_E C_E}{1 + s\dfrac{R_E(R_S + r_\pi)C_E}{R_S + r_\pi + (1+\beta)R_E}}$$

$$= \frac{-\beta R_C}{R_S + r_\pi + (1+\beta)R_E} \times \frac{1 + s\tau_A}{1 + s\tau_B}$$

$$\left|A_v(s)\right|_{\omega\to 0} = \frac{\beta R_C}{R_S + r_\pi + (1+\beta)R_E}$$

$$\left|A_v(s)\right|_{\omega\to \infty} = \frac{\beta R_C}{R_S + r_\pi}$$



## Bypass Capacitor Effects

The Bode plot:

$$|A_v(jf)|_{dB} = 20\log_{10} K + 20\log_{10}|1 + j2\pi f\tau_A| - 20\log_{10}|1 + j2\pi f\tau_B|$$

$$f_A = \frac{1}{2\pi\tau_A} \qquad \tau_A = R_E C_E$$

$$f_B = \frac{1}{2\pi\tau_B} \qquad \tau_B = \frac{R_E(R_S + r_\pi)C_E}{R_S + r_\pi + (1+\beta)R_E} \approx \frac{R_E(R_S + r_\pi)}{(1+\beta)R_E}C_E = \frac{R_S + r_\pi}{1+\beta}C_E$$

For typical parameter values, $\tau_A >> \tau_B \Rightarrow f_A << f_B$.

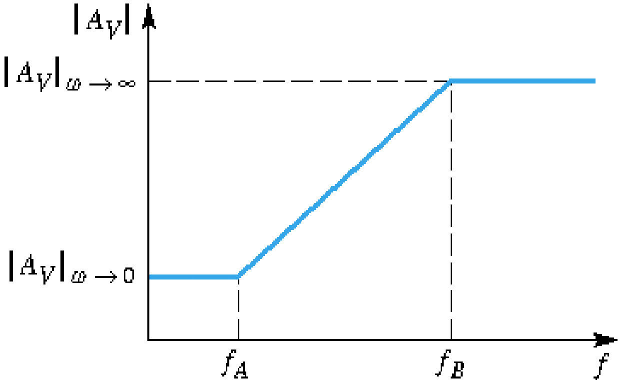
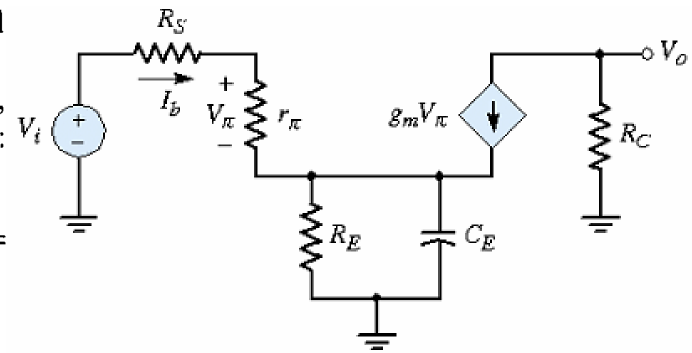



**Figure 7.24** Bode plot of the voltage gain magnitude for the circuit with an emitter bypass capacitor

**Example 7.8** **Objective:** Determine the corner frequencies and limiting horizontal asymptotes of a common-emitter circuit with an emitter bypass capacitor.

Consider the circuit in Figure 7.23(a) with parameters $R_E = 4\,\text{k}\Omega$, $R_C = 2\,\text{k}\Omega$, $R_S = 0.5\,\text{k}\Omega$, $C_E = 1\,\mu\text{F}$, $V^+ = 5\,\text{V}$, and $V^- = -5\,\text{V}$. The transistor parameters are: $\beta = 100$, $V_{BE}(\text{on}) = 0.7\,\text{V}$, and $r_o = \infty$.




**Solution:** From the dc analysis, we find the quiescent collector current as $I_{CQ} = 1.06\,\text{mA}$. The transconductance is

$$g_m = \frac{I_{CQ}}{V_T} = \frac{1.06}{0.026} = 40.8\,\text{mA/V}$$

and the input base resistance is

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{1.06} = 2.45\,\text{k}\Omega$$

The time constant $\tau_A$ is

$$\tau_A = R_E C_E = (4 \times 10^3)(1 \times 10^{-6}) = 4 \times 10^{-3}\,\text{s}$$

and the time constant $\tau_B$ is

$$\tau_B = \frac{R_E(R_S + r_\pi)C_E}{[R_S + r_\pi + (1+\beta)R_E]} = \frac{(4 \times 10^3)(0.5 \times 10^3 + 2.45 \times 10^3)(1 \times 10^{-6})}{[0.5 \times 10^3 + 2.45 \times 10^3 + (101)(4 \times 10^3)]}$$

or

$$\tau_B = 2.90 \times 10^{-5}\,\text{s}$$

The corner frequencies are then

$$f_A = \frac{1}{2\pi\tau_A} = \frac{1}{2\pi(4 \times 10^{-3})} = 39.8\,\text{Hz}$$

and

$$f_B = \frac{1}{2\pi\tau_B} = \frac{1}{2\pi(2.9 \times 10^{-5})} \Rightarrow 5.49\,\text{kHz}$$

The limiting low-frequency horizontal asymptote, given by Equation (7.60(a)) is

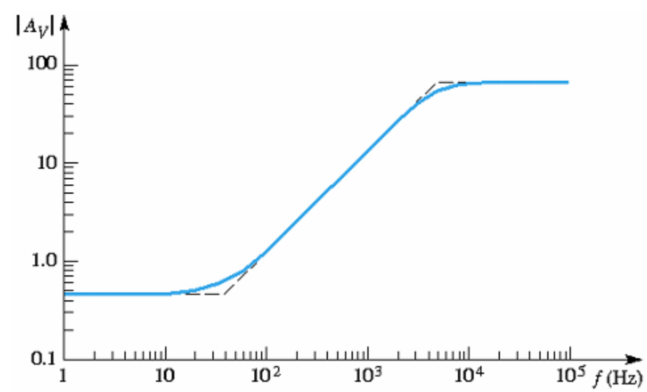
$$|A_v|_{\omega \to 0} = \frac{g_m r_\pi R_C}{[R_S + r_\pi + (1+\beta)R_E]} = \frac{(40.8)(2.45)(2)}{[0.5 + 2.45 + (101)(4)]}$$

or

$$|A_v|_{\omega \to 0} = 0.49$$

The limiting high-frequency horizontal asymptote, given by Equation (7.60(b)) is

$$|A_v|_{\omega \to \infty} = \frac{g_m r_\pi R_C}{R_S + r_\pi} = \frac{(40.8)(2.45)(2)}{0.5 + 2.45} = 67.8$$



# Expanded Hybrid-$\pi$ Equivalent Circuit for Frequency Response of BJTs

- Common-Emitter NPN BJT

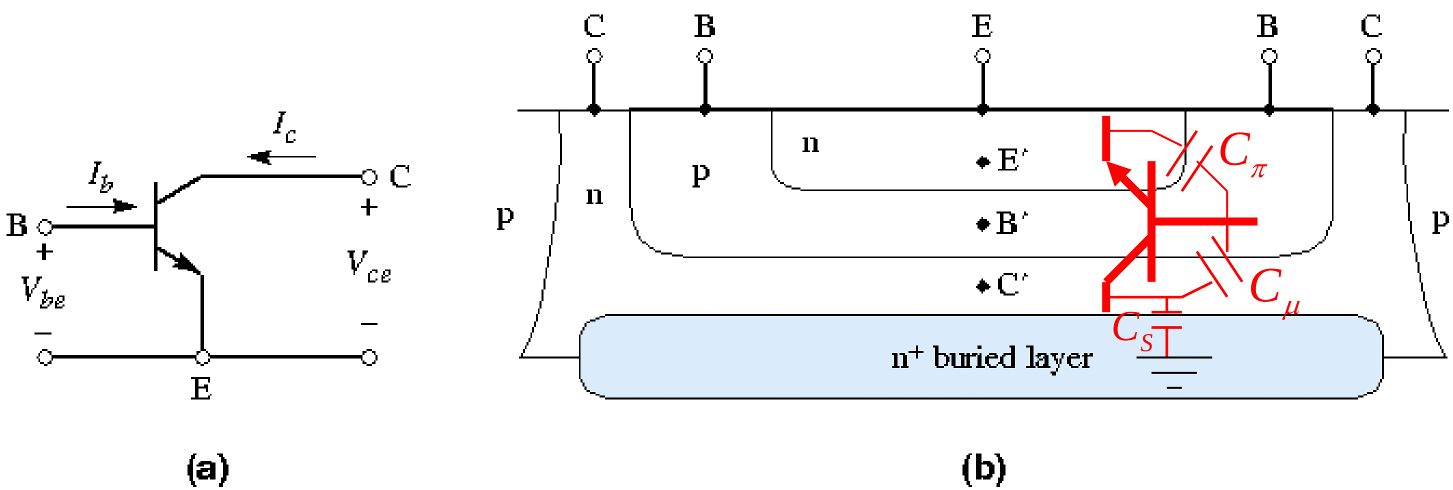



Figure 7.31 (a) Common-emitter npn bipolar transistor with small-signal currents and voltages and (b) cross section of an npn bipolar transistor, for the hybrid-$\pi$ model

# Expanded Hybrid-$\pi$ Equivalent Circuit for Frequency Response of BJTs

- Equivalent Circuit with Parasitic Capacitors

$r_b, r_{ex}, r_c$ : the series resistance between the external terminal and internal region

$r_\pi$ : forward-biased junction diffusion resistance

$r_\mu$ : reverse-biased junction diffusion resistance, on the order of MegaOhms and can be neglected

$C_\pi$ : forward-biased junction capacitance

$C_\mu$ : reverse-biased junction capacitance, $C_\mu < C_\pi$

$C_S$ : collector-substrate junction capacitance

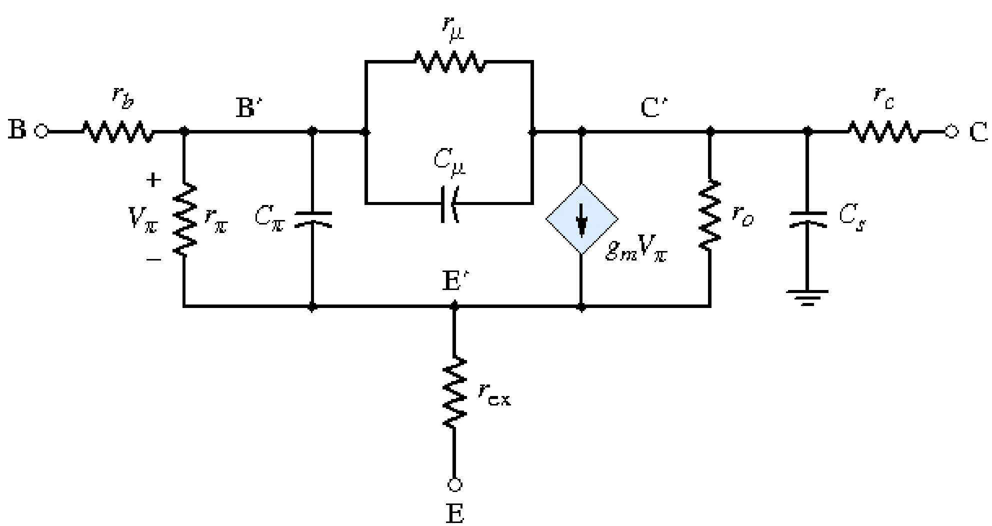



Figure 7.33 Hybrid-$\pi$ equivalent circuit

# Short-Circuit Current Gain

- Small-Signal Current Gain for Frequency Effects

$$I_b = V_\pi \left[ \frac{1}{r_\pi} + j\omega(C_\pi + C_\mu) \right] \qquad V_\pi = I_b \Big/ \left[ \frac{1}{r_\pi} + j\omega(C_\pi + C_\mu) \right]$$

$$\frac{V_\pi}{1/ j\omega C_\mu} + I_c = g_m V_\pi$$

$$I_c = V_\pi (g_m - j\omega C_\mu) = \frac{g_m - j\omega C_\mu}{\frac{1}{r_\pi} + j\omega(C_\pi + C_\mu)} I_b$$

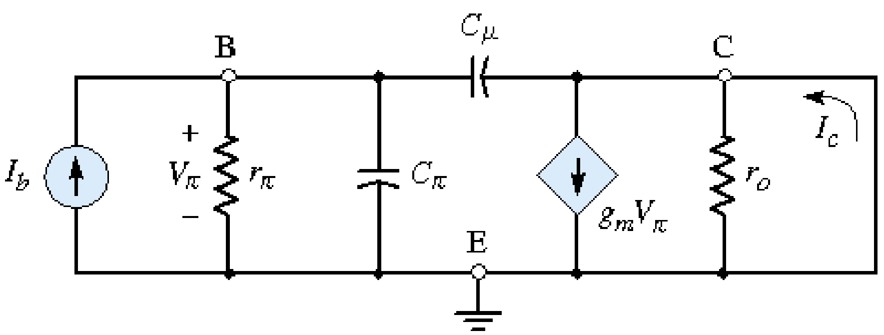
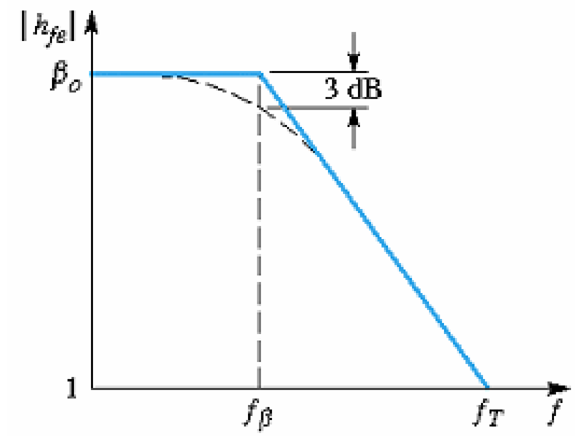



Figure 7.34 Simplified hybrid-$\pi$ equivalent circuit for determining the short-circuit current gain

# Short-Circuit Current Gain

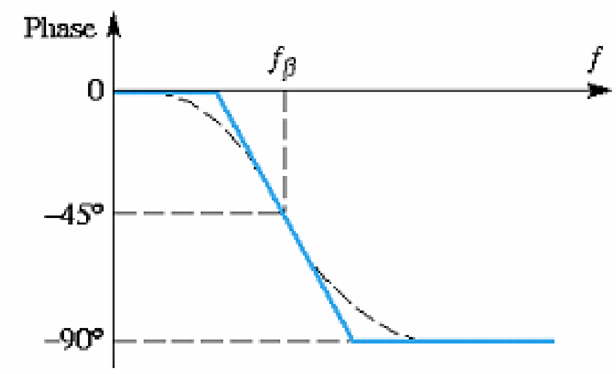
- For typical parameter values, $\omega C_\mu << g_m$

$$A_i = \frac{I_c}{I_b} \approx \frac{g_m}{\frac{1}{r_\pi} + j\omega(C_\pi + C_\mu)} = \frac{\beta}{1 + j\omega r_\pi (C_\pi + C_\mu)}$$




- Beta cut-off frequency: $f_\beta = \dfrac{1}{2\pi r_\pi (C_\pi + C_\mu)}$
- Cutoff Frequency

At the cutoff frequency $f_T$, $A_i$ goes to 1.




$$|A_i|_{f=f_T} = \left|\frac{\beta_0}{1 + j(f_T / f_\beta)}\right| = \frac{\beta_0}{\sqrt{1 + (f_T / f_\beta)^2}} = 1$$

$$1 \cong \frac{\beta_0}{\sqrt{(f_T / f_\beta)^2}} = \frac{\beta_0 f_\beta}{f_T}$$

$$f_T = \beta_0 f_\beta = \beta_0 \left[\frac{1}{2\pi r_\pi (C_\pi + C_\mu)}\right] = \frac{g_m}{2\pi (C_\pi + C_\mu)}$$ (Small Sizes => High Freq.)

**Example 7.9** **Objective:** Determine the 3 dB frequency of the short-circuit current gain of a bipolar transistor.

Consider a bipolar transistor with parameters $r_\pi = 2.6\,\text{k}\Omega$, $C_\pi = 2\,\text{pF}$, and $C_\mu = 0.1\,\text{pF}$.

**Solution:** From Equation (7.67), we have

$$f_\beta = \frac{1}{2\pi r_\pi(C_\pi + C_\mu)} = \frac{1}{2\pi(2.6 \times 10^3)(2 + 0.1)(10^{-12})}$$

or

$$f_\beta = 29.1\,\text{MHz}$$

**Example 7.10** **Objective:** Calculate the bandwidth $f_\beta$ and capacitance $C_\pi$ of a bipolar transistor.

Consider a transistor that has parameters $f_T = 500\,\text{MHz}$ at $I_C = 1\,\text{mA}$, $\beta_o = 100$, and $C_\mu = 0.3\,\text{pF}$.

**Solution:** From Equation (7.73), the bandwidth is

$$f_\beta = \frac{f_T}{\beta_o} = \frac{500}{100} = 5\,\text{MHz}$$

The transconductance is

$$g_m = \frac{I_C}{V_T} = \frac{1}{0.026} = 38.5\,\text{mA/V}$$

The $C_\pi$ capacitance is determined from Equation (7.72). We have

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

or

$$500 \times 10^6 = \frac{38.5 \times 10^{-3}}{2\pi(C_\pi + 0.3 \times 10^{-12})}$$



which yields $C_\pi = 12.0\,\text{pF}$.

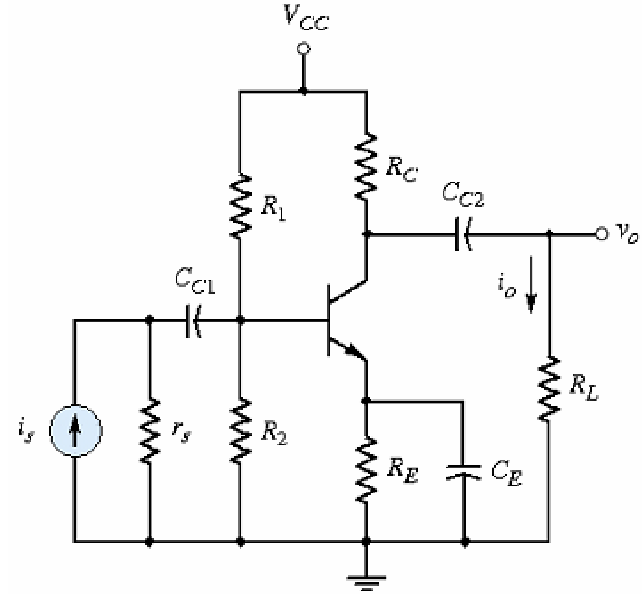
# Miller Effect

- Consider that the frequency is sufficiently high for the coupling and bypass capacitors to act as short circuit.

$$I_1 = \frac{V_\pi - V_o}{1/(j\omega C_\mu)} = (V_\pi - V_o)(j\omega C_\mu)$$

$$= g_m V_\pi + V_o/(R_C // R_L)$$

$$V_\pi(j\omega C_\mu - g_m) = V_o[1/(R_C // R_L) + j\omega C_\mu]$$




For typical parameter values,

$$g_m >> |j\omega C_\mu| \text{ and } 1/(R_C // R_L) >> |j\omega C_\mu|$$

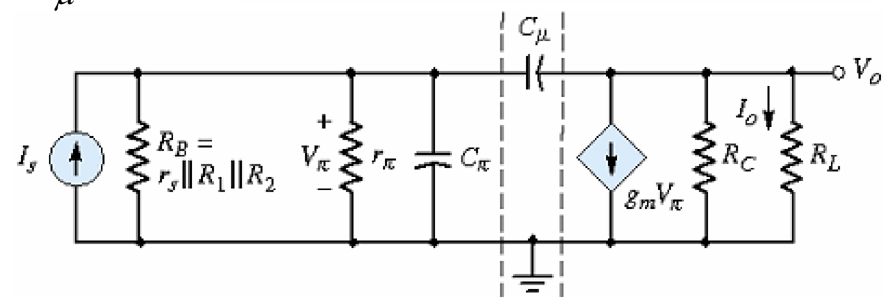
$$-g_m V_\pi \approx V_o/(R_C // R_L)$$

$$I_1 = (V_\pi - V_o)(j\omega C_\mu)$$

$$= V_\pi \cdot j\omega C_\mu[1 + g_m(R_C // R_L)]$$

$$= V_\pi \cdot Z_A$$

$$Z_A = j\omega C_\mu[1 + g_m(R_C // R_L)]$$



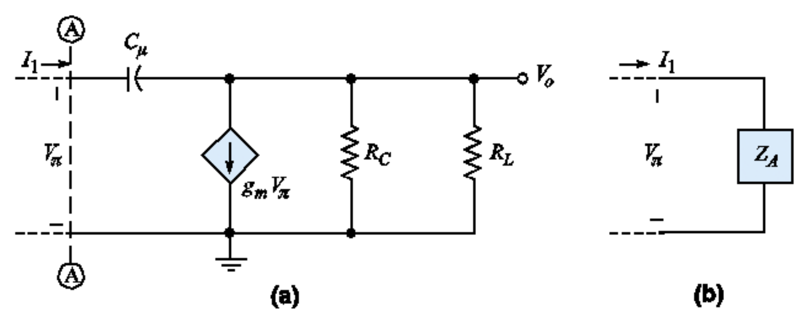




**Figure 7.38** (a) Output portion of small-signal equivalent circuit; (b) equivalent impedance of this portion of the circuit

# Miller Capacitance

- Miller Capacitance

$$C_M = C_\mu[1 + g_m(R_C // R_L)]$$

- Miller Effect: the multiplication effect of $C_\mu$
- Output Voltage

$$V_o \approx -g_m V_\pi (R_C // R_L)$$

$$V_\pi(j\omega C_\mu - g_m) = V_o[1/(R_C // R_L) + j\omega C_\mu]$$
$$\approx -V_\pi g_m$$
$$V_o = -\frac{g_m V_\pi}{1/(R_C // R_L) + j\omega C_\mu}$$
$$= -\frac{g_m V_\pi}{\dfrac{1}{R_C // R_L} + \dfrac{1}{1/j\omega C_\mu}}$$
$$= -g_m V_\pi \cdot \left[(R_C // R_L) // (1/j\omega C_\mu)\right]$$

- Input Capacitance

$$C_{in} = C_\pi + C_M$$

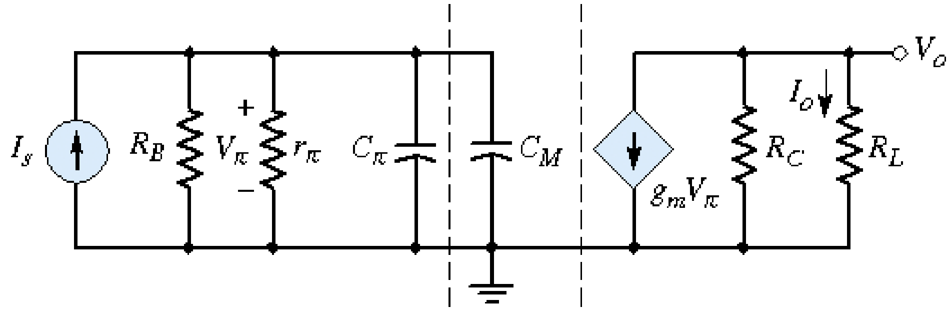
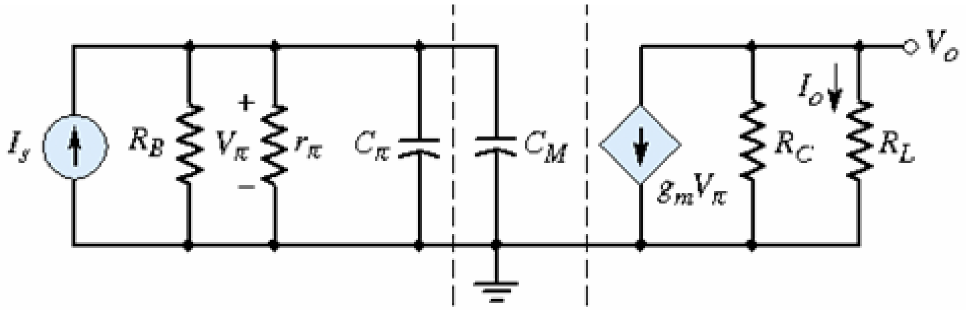



**Figure 7.39** Small-signal equivalent circuit, including the equivalent Miller capacitance

**Example 7.11** **Objective:** Determine the 3 dB frequency of the current gain for the circuit shown in Figure 7.39, both with and without the effect of $C_M$.

The circuit parameters are: $R_C = R_L = 4\,\text{k}\Omega$, $r_\pi = 2.6\,\text{k}\Omega$, $R_B = 200\,\text{k}\Omega$, $C_\pi = 4\,\text{pF}$, $C_\mu = 0.2\,\text{pF}$, and $g_m = 38.5\,\text{mA/V}$.




**Solution:** The output current can be written as

$$I_o = -(g_m V_\pi)\left(\frac{R_C}{R_C + R_L}\right)$$

Also, the input voltage is

$$V_\pi = I_s\left[R_B \| r_\pi \left\| \frac{1}{j\omega C_\pi} \right\| \frac{1}{j\omega C_M}\right] = I_s\left[\frac{R_B \| r_\pi}{1 + j\omega(R_B \| r_\pi)(C_\pi + C_M)}\right]$$

Therefore, the current gain is

$$A_i = \frac{I_o}{I_s} = -g_m\left(\frac{R_C}{R_C + R_L}\right)\left[\frac{R_B \| r_\pi}{1 + j\omega(R_B \| r_\pi)(C_\pi + C_M)}\right]$$

The 3 dB frequency is

$$f_{3\,\text{dB}} = \frac{1}{2\pi(R_B \| r_\pi)(C_\pi + C_M)}$$

Neglecting the effect of $C_\mu (C_M = 0)$, we find that

$$f_{3\,\text{dB}} = \frac{1}{2\pi[(200 \times 10^3) \| (2.6 \times 10^3)](4 \times 10^{-12})} \Rightarrow 15.5\,\text{MHz}$$

The Miller capacitance is

$$C_M = C_\mu[1 + g_m(R_C \| R_L)] = (0.2)[1 + (38.5)(4\|4)] = 15.6\,\text{pF}$$

Taking into account the Miller capacitance, the 3 dB frequency is

$$f_{3\,\text{dB}} = \frac{1}{2\pi(R_B \| r_\pi)(C_\pi + C_M)} = \frac{1}{2\pi[(200 \times 10^3) \| (2.6 \times 10^3)][4 + 15.6](10^{-12})}$$

or

$$f_{3\,\text{dB}} = 3.16\,\text{MHz}$$

# FET High Frequency Equivalent Circuit

- N-channel Common Source MOSFET

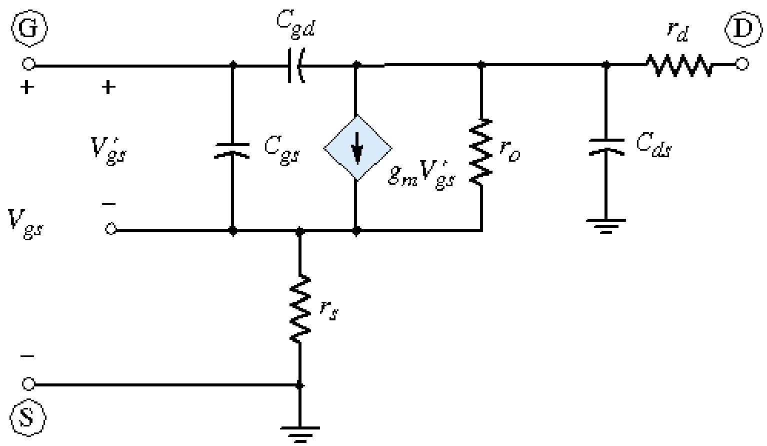
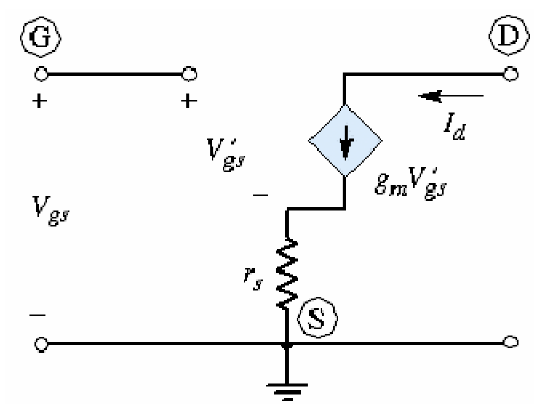



**Figure 7.41** Equivalent circuit of the n-channel common-source MOSFET

- Effect of the Source Resistance

$$V_{gs} = V'_{gs} + g_m V'_{gs} r_s = (1 + g_m r_s) V'_{gs}$$

$$I_d = g_m V'_{gs} = \left( \frac{g_m}{1 + g_m r_s} \right) V_{gs} = g'_m V_{gs}$$



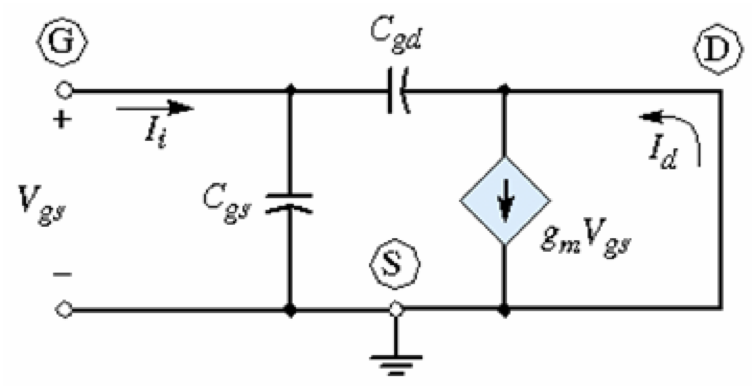
# Unity-Gain Bandwidth

$$I_i = V_{gs}/(1/j\omega C_{gs})) + V_{gs}/(1/j\omega C_{gd})$$
$$= V_{gs}[j\omega(C_{gs} + C_{gd})]$$

$$V_{gs}/(1/j\omega C_{gd}) + I_d = g_m V_{gs}$$

$$I_d = V_{gs}(g_m - j\omega C_{gd})$$
$$= \frac{g_m - j\omega C_{gd}}{j\omega(C_{gs} + C_{gd})} I_i$$

$$A_i = \frac{I_d}{I_i} \approx \frac{g_m}{j\omega(C_{gs} + C_{gd})}$$

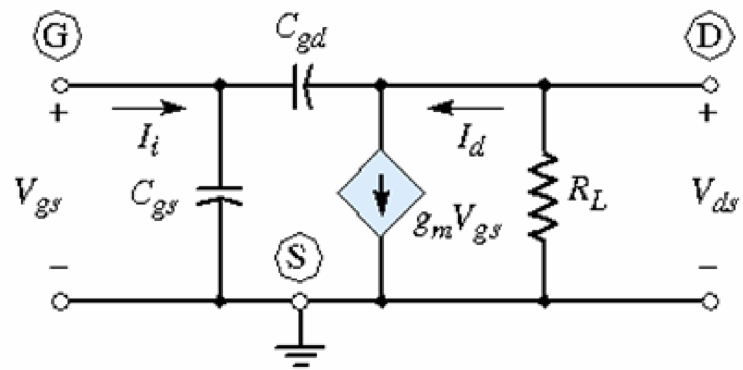



The unity-gain frequency $f_T$ is defined as the frequency at which the magnitude of the short-circuit current gain goes to 1.

$$1 = \left| \frac{g_m}{j2\pi f_T(C_{gs} + C_{gd})} \right| \qquad f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

# Miller Effect and Miller Capacitance

$$I_i = j\omega C_{gs}V_{gs} + j\omega C_{gd}(V_{gs} - V_{ds})$$
$$= j\omega(C_{gs} + C_{gd})V_{gs} - j\omega C_{gd}V_{ds}$$




$$j\omega C_{gd}(V_{gs} - V_{ds}) = g_m V_{gs} + V_{ds} / R_L$$

$$(j\omega C_{gd} - g_m)V_{gs} = (j\omega C_{gd} + 1/R_L)V_{ds}$$
$$\approx V_{ds} / R_L$$

$$I_i = j\omega(C_{gs} + C_{gd})V_{gs} - j\omega C_{gd}V_{ds}$$
$$= j\omega(C_{gs} + C_{gd})V_{gs} - j\omega C_{gd}(j\omega C_{gd} - g_m)R_L V_{gs}$$
$$= j\omega[C_{gs} + C_{gd}(1 + g_m R_L - j\omega C_{gd}R_L)]V_{gs}$$
$$\approx j\omega[C_{gs} + C_{gd}(1 + g_m R_L)]V_{gs}$$

The Miller Capacitance: $C_M = C_{gd}(1 + g_m R_L)$

# Cutoff Frequency with Load Resistor

$$I_i = j\omega(C_{gs} + C_M)V_{gs}$$

$$I_d = g_m V_{gs} = \frac{g_m}{j\omega(C_{gs} + C_M)} I_i$$

$$|A_i| = \frac{g_m}{2\pi f(C_{gs} + C_M)}$$

$$C_M = C_{gd}(1 + g_m R_L)$$

If $R_L = 0$, then $C_M = C_{gd}$.

Cutoff Frequency: $f_T = \dfrac{g_m}{2\pi(C_{gs} + C_M)}$

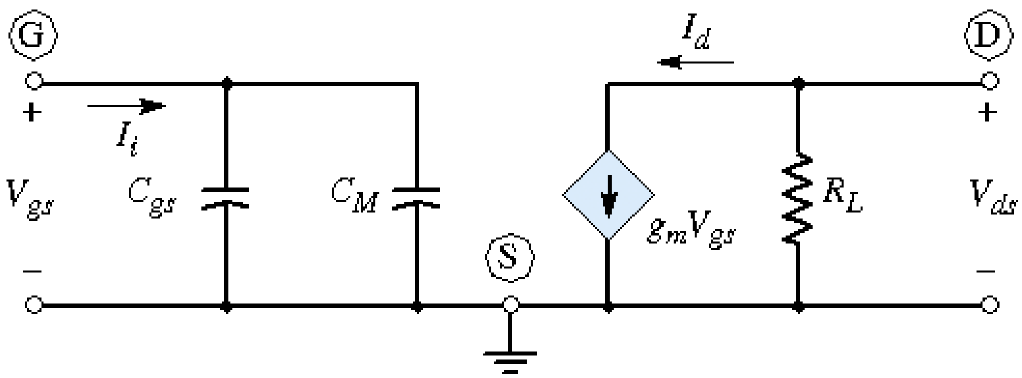



**Figure 7.45** MOSFET high-frequency circuit, including the equivalent Miller capacitance

**Example 7.12** **Objective:** Determine the unity-gain bandwidth of an FET.

Consider an n-channel MOSFET with parameters $K_n = 0.25\,\text{mA/V}^2$, $V_{TN} = 1\,\text{V}$, $\lambda = 0$, $C_{gd} = 0.04\,\text{pF}$, and $C_{gs} = 0.2\,\text{pF}$. Assume the transistor is biased at $V_{GS} = 3\,\text{V}$.

**Solution:** The transconductance is

$$g_m = 2K_n(V_{GS} - V_{TN}) = 2(0.25)(3 - 1) = 1\,\text{mA/V}$$

From Equation (7.90), the unity-gain bandwidth, or frequency, is

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})} = \frac{10^{-3}}{2\pi(0.2 + 0.04) \times 10^{-12}} = 6.63 \times 10^8\,\text{Hz}$$

or

$$f_T = 663\,\text{MHz}$$

**Example 7.13** **Objective:** Determine the Miller capacitance and cutoff frequency of an FET circuit.

The n-channel MOSFET described in Example 7.12 is biased at the same current, and a $10\,\text{k}\Omega$ load is connected to the output.

**Solution:** From Example 7.12, the transconductance is $g_m = 1\,\text{mA/V}$. The Miller capacitance is therefore

$$C_M = C_{gd}(1 + g_m R_L) = (0.04)[1 + (1)(10)] = 0.44\,\text{pF}$$

From Equation (7.99), the cutoff frequency is

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_M)} = \frac{10^{-3}}{2\pi(0.2 + 0.44) \times 10^{-12}} = 2.49 \times 10^8\,\text{Hz}$$

or

$$f_T = 249\,\text{MHz}$$

# Common-Emitter Amplifier

- High-Frequency Equivalent Model

$$C_M = C_\mu(1+g_m(R_L // R_C // r_o))$$

$$f_H = \frac{1}{2\pi\tau_P}$$

$$|A_v| = g_m R'_L \left[\frac{r_\pi // R_B}{r_\pi // R_B + R_S}\right]$$

$$\tau_P = R_{eq}C_{eq}$$

$$R_{eq} = r_\pi // R_B // R_S$$

$$C_{eq} = C_\pi + C_M$$

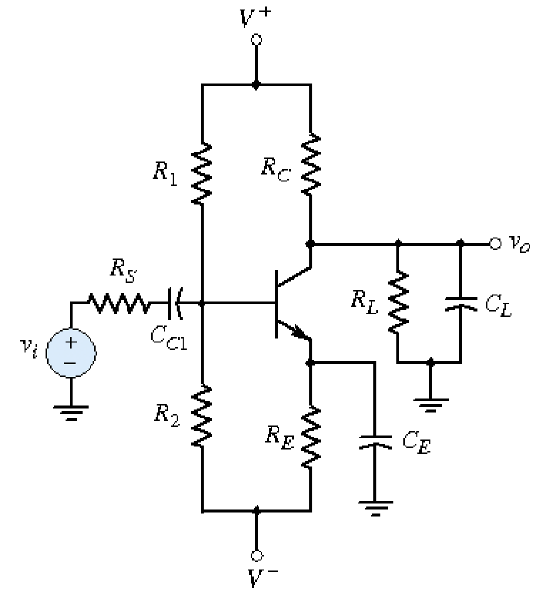



Figure 7.47 Common-emitter amplifier

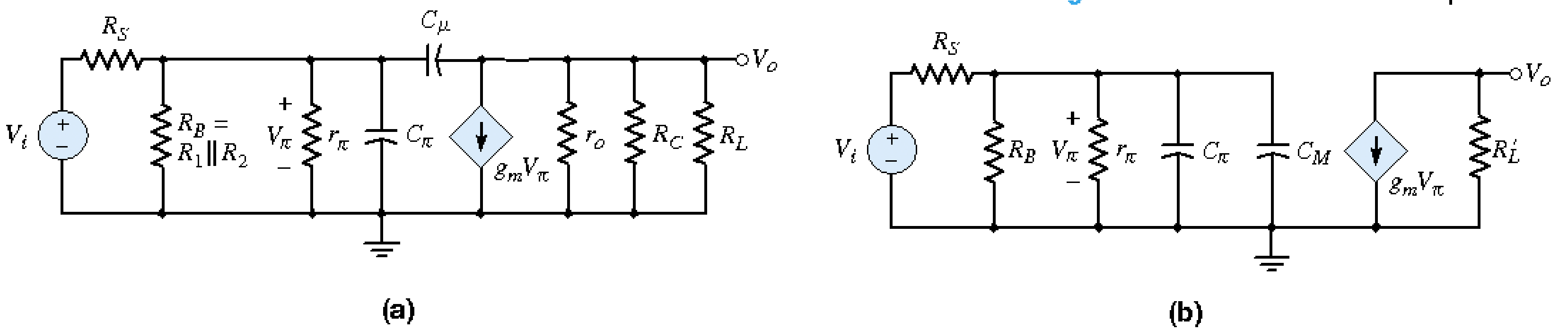



Figure 7.48 (a) High-frequency equivalent circuit of common-emitter amplifier; (b) high-frequency equivalent circuit of common-emitter amplifier, including the Miller capacitance

**Example 7.14** **Objective:** Determine the upper corner frequency and midband gain of a common-emitter circuit.

For the circuit in Figure 7.47, the parameters are: $V^+ = 5\,\text{V}$, $V^- = -5\,\text{V}$, $R_S = 0.1\,\text{k}\Omega$, $R_1 = 40\,\text{k}\Omega$, $R_2 = 5.72\,\text{k}\Omega$, $R_E = 0.5\,\text{k}\Omega$, $R_C = 5\,\text{k}\Omega$, and $R_L = 10\,\text{k}\Omega$. The transistor parameters are: $\beta = 150$, $V_{BE}(\text{on}) = 0.7\,\text{V}$, $V_A = \infty$, $C_\pi = 35\,\text{pF}$, and $C_\mu = 4\,\text{pF}$.

**Solution:** From a dc analysis, we find that $I_{CQ} = 1.02\,\text{mA}$. The small-signal parameters are therefore

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(150)(0.026)}{1.02} = 3.82\,\text{k}\Omega$$

and

$$g_m = \frac{I_{CQ}}{V_T} = \frac{1.02}{0.026} = 39.2\,\text{mA/V}$$

The Miller capacitance is then

$$C_M = C_\mu(1 + g_m R_L') = C_\mu[1 + g_m(R_C \| R_L)]$$

or

$$C_M = (4)[1 + (39.2)(5\|10)] = 527\,\text{pF}$$

and the upper 3 dB frequency is therefore

$$f_H = \frac{1}{2\pi[r_\pi \| R_B \| R_S](C_\pi + C_M)}$$
$$= \frac{1}{2\pi[3.82\|40\|5.72\|0.1](10^3)(35 + 527)(10^{-12})}$$

or

$$f_H = 2.96\,\text{MHz}$$

Finally, the midband gain is

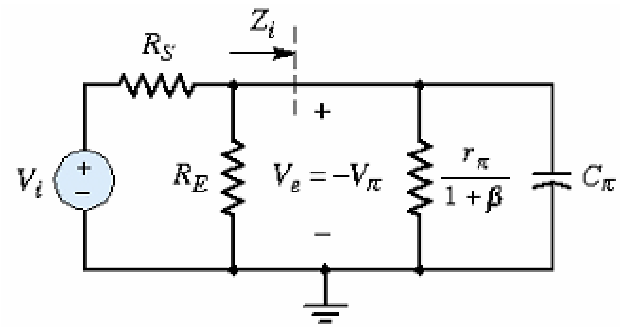
$$|A_v|_M = g_m R_L' \left[\frac{R_B \| r_\pi}{R_B \| r_\pi + R_S}\right]$$
$$= (39.2)(5\|10)\left[\frac{40\|5.72\|3.82}{40\|5.72\|3.82 + 0.1}\right]$$

or

$$|A_v|_M = 125$$

# Common-Base Amplifier

- Equivalent Input Circuit




$$I_e + V_\pi / r_\pi + sC_\pi V_\pi + g_m V_\pi = 0$$

$$I_e = V_e(1/r_\pi + sC_\pi + g_m), \quad V_\pi = -V_e$$

$$\frac{1}{Z_i} = \frac{I_e}{V_e} = \frac{1+\beta}{r_\pi} + sC_\pi$$

$$Z_i = \frac{r_\pi}{1+\beta} // \frac{1}{sC_\pi}$$

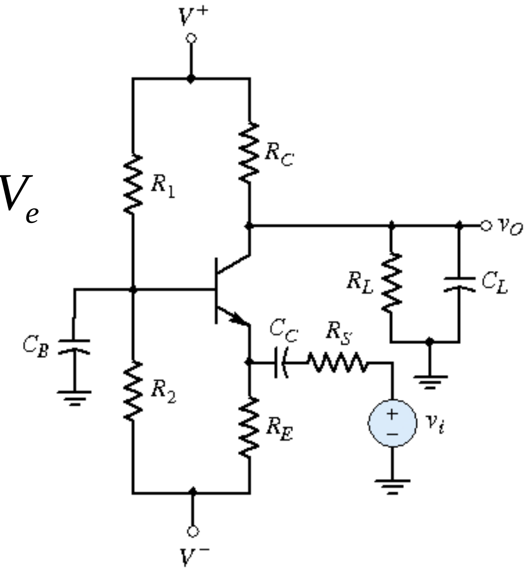
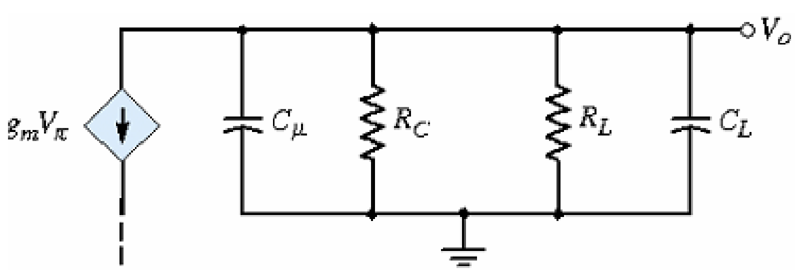
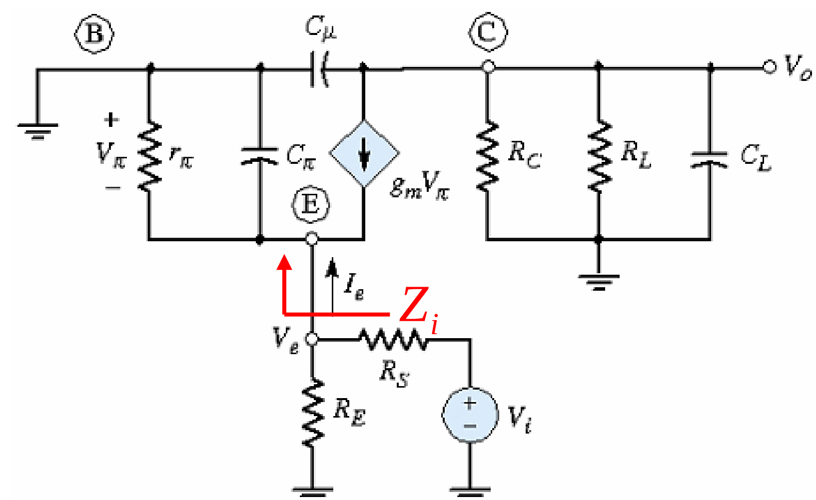



**Figure 7.51** Common-base amplifier

$$f_{H\pi} = \frac{1}{2\pi\tau_{p\pi}} \qquad \tau_{p\pi} = \left[\frac{r_\pi}{1+\beta} // R_E // R_S\right] C_\pi$$

- Equivalent Output Circuit




$$f_{H\mu} = \frac{1}{2\pi\tau_{p\mu}} \qquad \tau_{p\mu} = (R_C // R_L) C_\mu$$

**Example 7.15** **Objective:** Determine the upper corner frequencies and midband gain of a common-base circuit.

Consider the circuit shown in Figure 7.51 with circuit parameters $V^+ = 5\,\text{V}$, $V^- = -5\,\text{V}$, $R_S = 0.1\,\text{k}\Omega$, $R_1 = 40\,\text{k}\Omega$, $R_2 = 5.72\,\text{k}\Omega$, $R_E = 0.5\,\text{k}\Omega$, $R_C = 5\,\text{k}\Omega$, and $R_L = 10\,\text{k}\Omega$. (These are the same values as those used for the common-emitter circuit in Example 7.14.) The transistor parameters are: $\beta = 150$, $V_{BE}(\text{on}) = 0.7\,\text{V}$, $V_A = \infty$, $C_\pi = 35\,\text{pF}$, and $C_\mu = 4\,\text{pF}$.

**Solution:** The dc analysis is the same as in Example 7.14; therefore, $I_{CQ} = 1.02\,\text{mA}$, $g_m = 39.2\,\text{mA/V}$, and $r_\pi = 3.82\,\text{k}\Omega$. The time constant associated with $C_\pi$ is

$$\tau_{P\pi} = \left[\left(\frac{r_\pi}{1+\beta}\right)\|R_E\|R_S\right]\cdot C_\pi$$

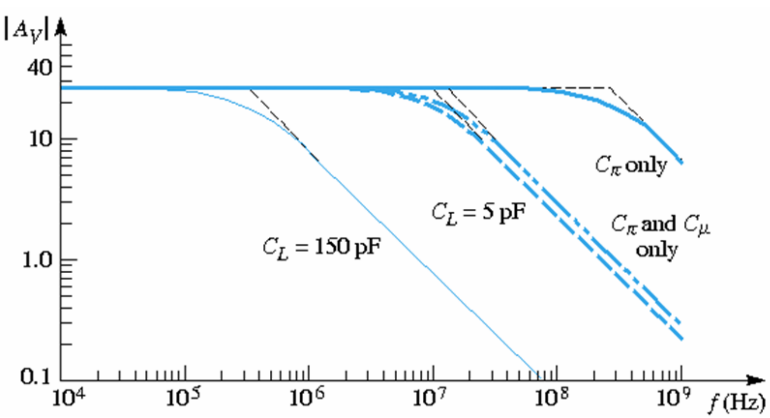
$$= \left[\left(\frac{3.82}{151}\right)\|(0.5)\|(0.1)\right]\times 10^3(35\times 10^{-12})$$

or

$$\tau_{P\pi} = 0.679\,\text{ns}$$

The upper 3 dB frequency corresponding to $C_\pi$ is therefore

$$f_{H\pi} = \frac{1}{2\pi\tau_{P\pi}} = \frac{1}{2\pi(0.679\times 10^{-9})} \Rightarrow 234\,\text{MHz}$$




The time constant associated with $C_\mu$ in the output portion of the circuit is

$$\tau_{P\mu} = [R_C\|R_L]\cdot C_\mu = [5\|10]\times 10^3(4\times 10^{-12}) \Rightarrow 13.3\,\text{ns}$$

The upper 3 dB frequency corresponding to $C_\mu$ is therefore

$$f_{H\mu} = \frac{1}{2\pi\tau_{P\mu}} = \frac{1}{2\pi(13.3\times 10^{-9})} \Rightarrow 12.0\,\text{MHz}$$

So in this case, $f_{H\mu}$ is the dominant pole frequency.

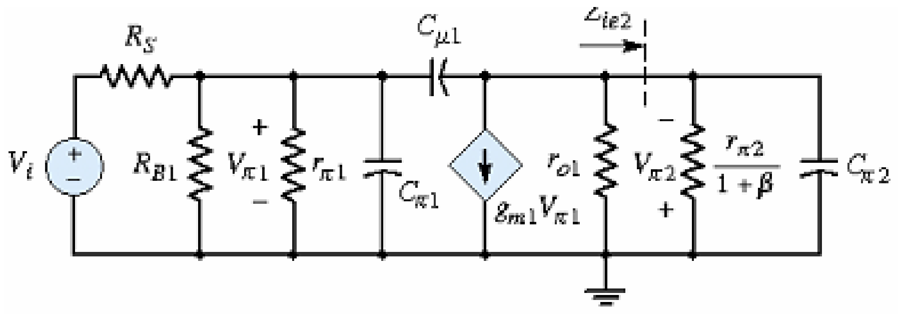
The magnitude of the midband voltage gain is

$$|A_v|_M = g_m(R_C\|R_L)\left[\frac{R_E\left\|\left(\frac{r_\pi}{1+\beta}\right)\right.}{R_E\left\|\left(\frac{r_\pi}{1+\beta}\right)\right.+R_S}\right]$$

$$= (39.2)(5\|10)\left[\frac{0.5\left\|\left(\frac{3.82}{151}\right)\right.}{0.5\left\|\left(\frac{3.82}{151}\right)\right.+0.1}\right] = 25.3$$

# Cascode Circuit

- Input Impedance to the emitter of Q2



$$Z_{ie2} = \frac{r_{\pi 2}}{1+\beta} // \frac{1}{sC_{\pi 2}}$$

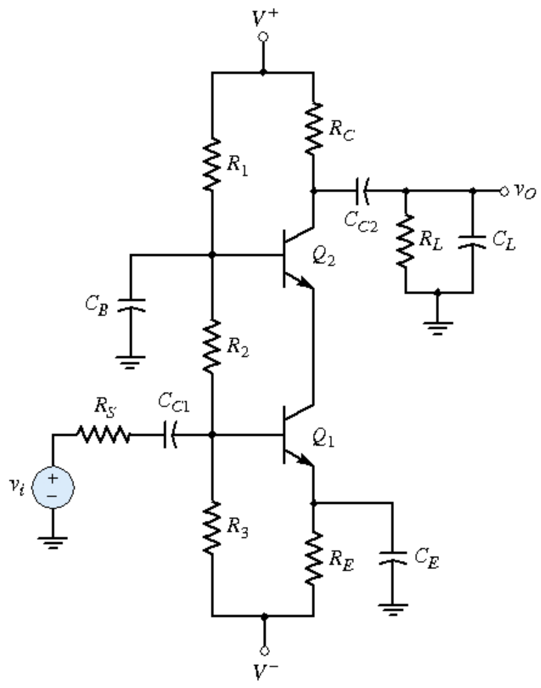
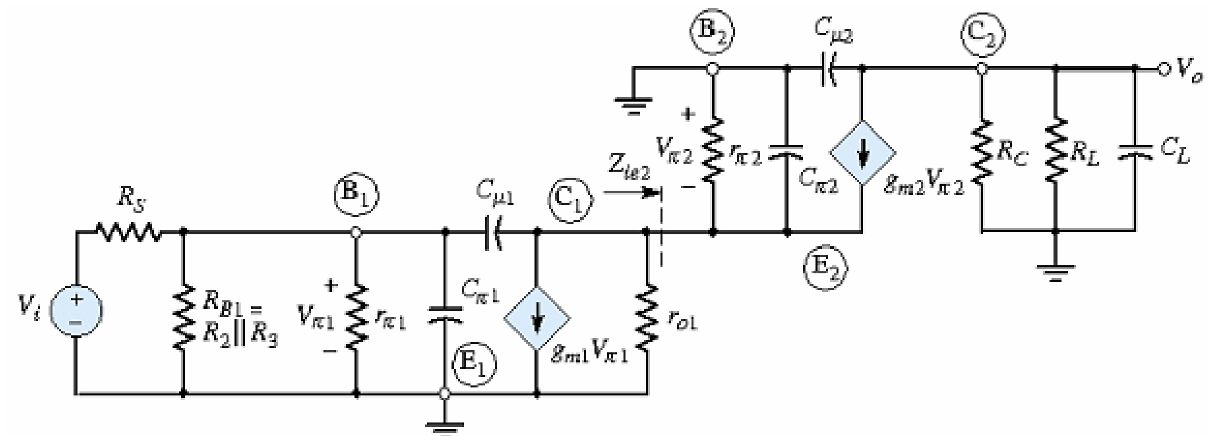
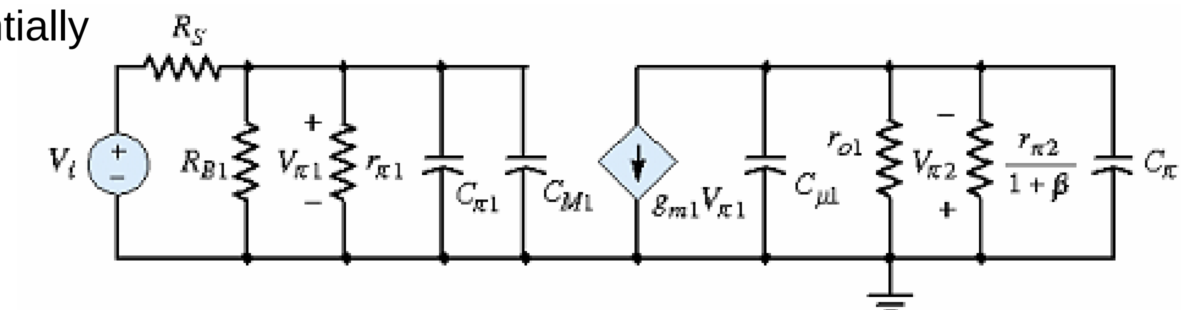


**Figure 7.54** Cascode circuit



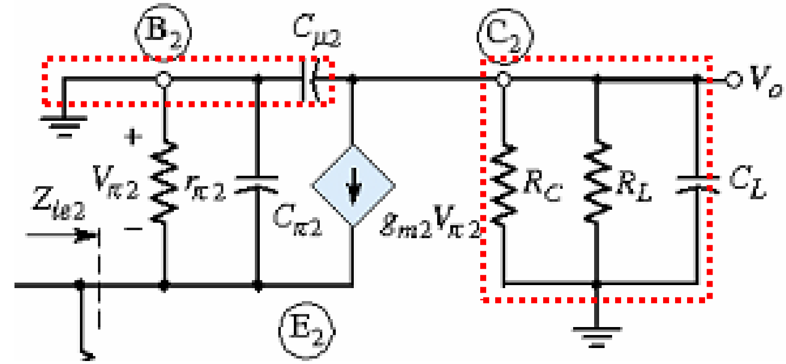
# Cascode Circuit

Q1 and Q2 are biased with essentially the same current,




$$C_{M1} = C_{\mu 1}\left(1 + g_{m1}\frac{r_{\pi 2}}{1+\beta}\right)$$

$$\approx C_{\mu 1}\left(1 + \frac{g_{m2} r_{\pi 2}}{1+\beta}\right) \approx 2C_{\mu 1} \qquad g_{m1} \approx g_{m2}$$




$$\tau_{p\pi 1} = (R_S \,//\, R_{B1} \,//\, r_{\pi 1})(C_{\pi 1} + 2C_{\mu 1})$$

$$\tau_{p\pi 2} = (C_{\mu 1} + C_{\pi 2})\left(r_{o1} \,//\, \frac{r_{\pi 2}}{1+\beta}\right) \approx (C_{\mu 1} + C_{\pi 2}) \cdot \frac{r_{\pi 2}}{1+\beta}$$

$$\tau_{P\pi 1} > \tau_{p\pi 2}, \quad f_{H\pi 1} < f_{H\pi 2}$$

$$f_{H\pi} = f_{H\pi 1} = \frac{1}{2\pi \tau_{p\pi 1}}$$

---

$$\tau_{p\mu} = (R_C \,//\, R_L) C_{\mu 2} \qquad C_{\mu 2} >> C_L$$

$$f_{H\mu} = f_{H\mu} = \frac{1}{2\pi \tau_{p\mu}}$$

$$V_o = -g_{m2}(R_C \,//\, R_L) V_{\pi 2}$$

$$= -g_{m2}(R_C \,//\, R_L) \cdot g_{m1}\frac{r_{\pi 2}}{1+\beta} V_{\pi 1}$$

$$= \underbrace{-g_{m2}(R_C \,//\, R_L) \cdot \frac{R_{B1} \,//\, r_{\pi 1}}{R_{B1} \,//\, r_{\pi 1} + R_S}}_{A_v} V_i$$

**Example 7.16** **Objective:** Determine the 3 dB frequencies and midband gain of a cascode circuit.

For the circuit in Figure 7.54, the parameters are: $V^+ = 10\,\text{V}$, $V^- = -10\,\text{V}$, $R_S = 0.1\,\text{k}\Omega$, $R_1 = 42.5\,\text{k}\Omega$, $R_2 = 20.5\,\text{k}\Omega$, $R_3 = 28.3\,\text{k}\Omega$, $R_E = 5.4\,\text{k}\Omega$, $R_C = 5\,\text{k}\Omega$, $R_L = 10\,\text{k}\Omega$, and $C_L = 0$. The transistor parameters are: $\beta = 150$, $V_{BE}(\text{on}) = 0.7\,\text{V}$, $V_A = \infty$, $C_\pi = 35\,\text{pF}$, and $C_\mu = 4\,\text{pF}$.

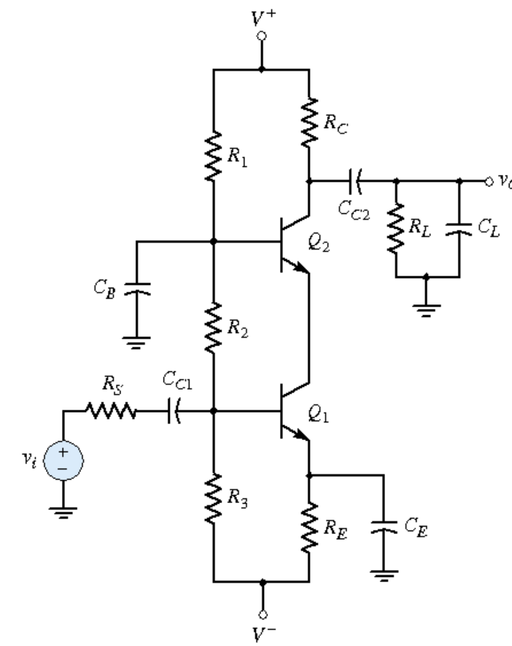



**Figure 7.54** Cascode circuit

**Solution:** Since $\beta$ is large for each transistor, the quiescent collector current is essentially the same in each transistor and is $I_{CQ} = 1.02\,\text{mA}$. The small-signal parameters are: $r_{\pi 1} = r_{\pi 2} \equiv r_\pi = 3.82\,\text{k}\Omega$ and $g_{m1} = g_{m2} \equiv g_m = 39.2\,\text{mA/V}$.

From Equation (7.112(a)), the time consant related to the input portion of the circuit is

$$\tau_{P\pi} = [R_S \| R_{B1} \| r_{\pi 1}](C_{\pi 1} + C_{M1})$$

Since $R_{B1} = R_2 \| R_3$ and $C_{M1} = 2\,C_{\mu 1}$, then

$$\tau_{P\pi} = [(0.1)\|20.5\|28.3\|3.82] \times 10^3 [35 + 2(4)] \times 10^{-12} \Rightarrow 4.16\,\text{ns}$$

The corresponding 3 dB frequency is

$$f_{H\pi} = \frac{1}{2\pi\tau_{P\pi}} = \frac{1}{2\pi(4.16 \times 10^{-9})} \Rightarrow 38.3\,\text{MHz}$$

From Equation (7.113(a)), the time constant of the output portion of the circuit is

$$\tau_{P\mu} = [R_C \| R_L] C_{\mu 2} = [5\|10] \times 10^3 (4 \times 10^{-12}) \Rightarrow 13.3\,\text{ns}$$

and the corresponding 3 dB frequency is

$$f_{H\mu} = \frac{1}{2\pi\tau_{P\mu}} = \frac{1}{2\pi(13.3 \times 10^{-9})} \Rightarrow 12\,\text{MHz}$$

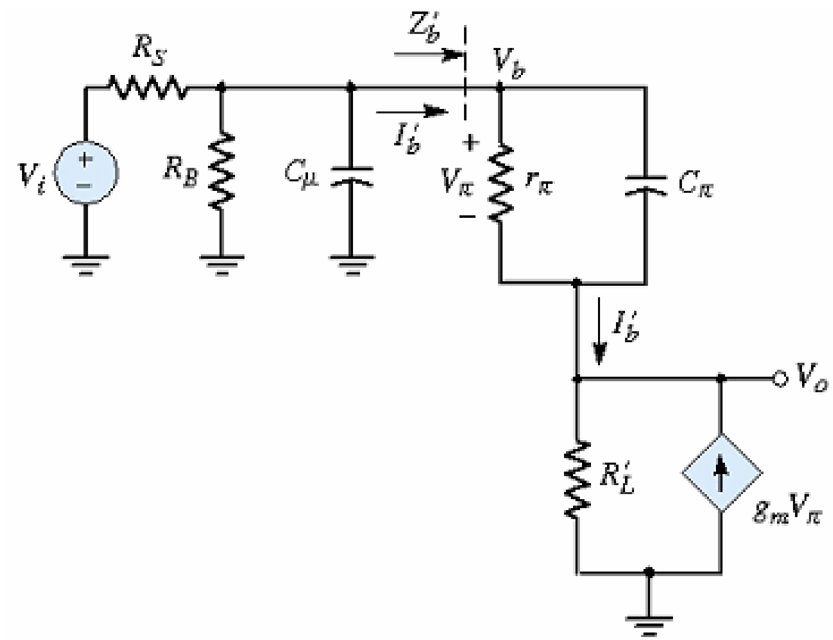
From Equation (7.118), the midband voltage gain is

$$|A_v|_M = g_{m2}(R_C \| R_L)\left[\frac{R_{B1} \| r_{\pi 1}}{R_{B1} \| r_{\pi 1} + R_S}\right]$$

$$= (39.2)(5\|10)\left[\frac{(20.5\|28.3\|3.82)}{(20.5\|28.3\|3.82) + (0.1)}\right] = 126$$

# Emitter-Follower Circuit

- The arranged high-frequency equivalent model



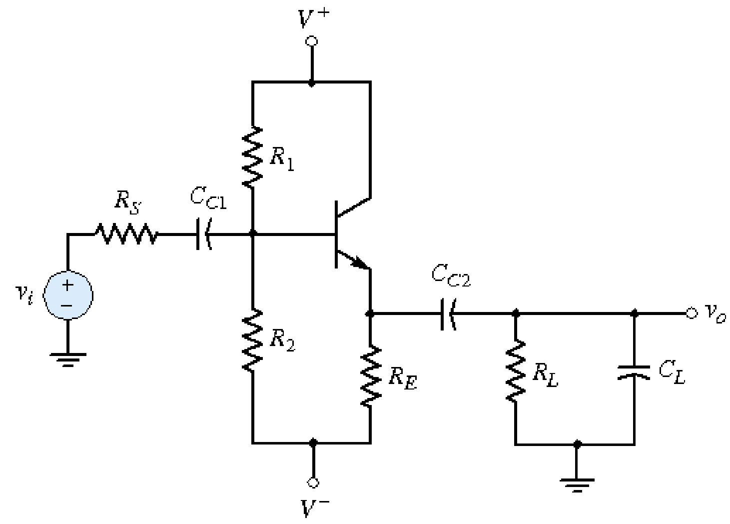
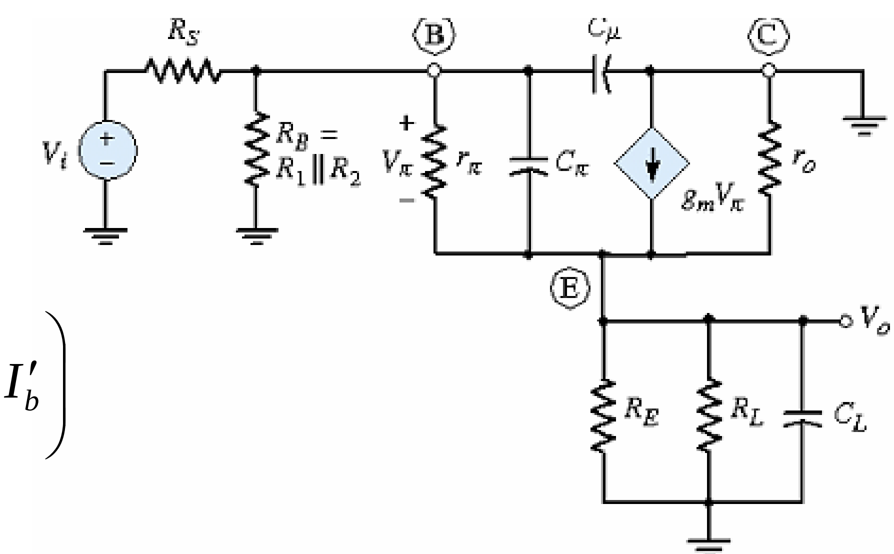




**Figure 7.57** Emitter-follower circuit




$$V_b = I_b'\left(r_\pi // \frac{1}{sC_\pi}\right) + R_L'\left(I_b' + g_m\left(r_\pi // \frac{1}{sC_\pi}\right)I_b'\right)$$

# Emitter-Follower Circuit

$$Z_b' = \frac{V_b}{I_b} = r_\pi // \frac{1}{sC_\pi} + R_L' + g_m R_L' \left( r_\pi // \frac{1}{sC_\pi} \right)$$

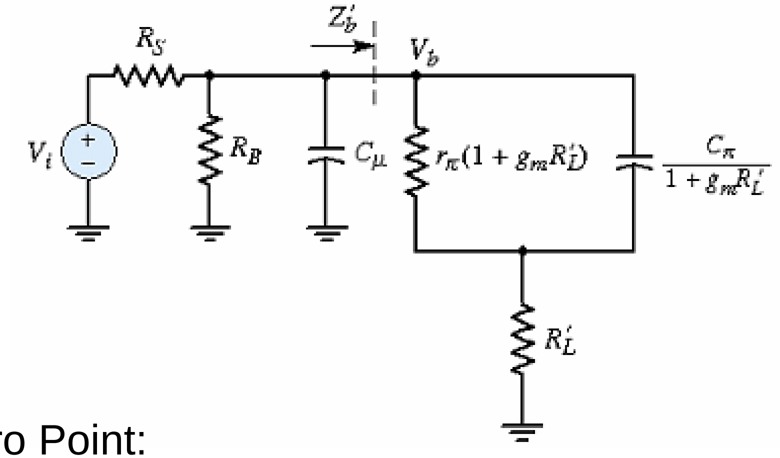
$$= R_L' + (1 + g_m R_L') \left( r_\pi // \frac{1}{sC_\pi} \right)$$

$$= r_\pi (1 + g_m R_L') // \frac{1}{sC_\pi / (1 + g_m R_L')} + R_L'$$

$$r_\pi (1 + g_m R_L') = r_\pi + \beta R_L' >> R_L'$$

$$\tau_p \approx [R_S // R_B // r_\pi (1 + g_m R_L')] \left( C_\mu + \frac{C_\pi}{1 + g_m R_L'} \right)$$

$$f_H = \frac{1}{2\pi\tau_P}$$ (3dB frequency or pole)




Zero Point:

$$V_o = g_m V_\pi R_L' + I_b' R_L' = \left( g_m + \frac{1}{r_\pi // (1/sC_\pi)} \right) \cdot R_L' V_\pi$$

$$= \left( g_m + \frac{1}{r_\pi} + sC_\pi \right) \cdot R_L' V_\pi$$

$$= \left( 1 + sC_\pi \frac{r_\pi}{1 + \beta} \right) \cdot \frac{1 + \beta}{r_\pi} \cdot R_L' V_\pi$$

$$1 + sC_\pi \left( \frac{r_\pi}{1 + \beta} \right) = 0 \quad \Rightarrow f_0 = \frac{1}{2\pi C_\pi \left( \frac{r_\pi}{1 + \beta} \right)}$$

**Example 7.17** **Objective:** Determine the frequency of a zero and a pole in the high-frequency response of an emitter follower.

Consider the emitter-follower circuit in Figure 7.57 with parameters $V^+ = 5\,\text{V}$, $V^- = -5\,\text{V}$, $R_S = 0.1\,\text{k}\Omega$, $R_1 = 40\,\text{k}\Omega$, $R_2 = 5.72\,\text{k}\Omega$, $R_E = 0.5\,\text{k}\Omega$, and $R_L = 10\,\text{k}\Omega$. The transistor parameters are: $\beta = 150$, $V_{BE}(\text{on}) = 0.7\,\text{V}$, $V_A = \infty$, $C_\pi = 35\,\text{pF}$, and $C_\mu = 4\,\text{pF}$.

**Solution:** As in previous examples, the dc analysis yields $I_{CQ} = 1.02\,\text{mA}$. Therefore, $g_m = 39.2\,\text{mA/V}$ and $r_\pi = 3.82\,\text{k}\Omega$.

From Equation (7.125), the zero occurs at

$$f_o = \frac{1}{2\pi C_\pi\left(\dfrac{r_\pi}{1+\beta}\right)} = \frac{1}{2\pi(35\times10^{-12})\left(\dfrac{3.82\times10^3}{151}\right)} \Rightarrow 180\,\text{MHz}$$

To determine the time constant for the high-frequency pole calculation, we know that

$$1 + g_m R_L' = 1 + g_m(R_E \| R_L) = 1 + (39.2)(0.5\|10) = 19.7$$

and

$$R_B = R_1 \| R_2 = 40 \| 5.72 = 5\,\text{k}\Omega$$

The time constant is therefore

$$\tau_P = [R_S \| R_B \| (1 + g_m R_L')r_\pi]\left(C_\mu + \frac{C_\pi}{1 + g_m R_L'}\right)$$

$$= [(0.1)\|5\|(19.7)(3.82)]\times10^3\left(4 + \frac{35}{19.7}\right)\times10^{-12} \Rightarrow 0.566\,\text{ns}$$

The 3 dB frequency (or pole) is then

$$f_H = \frac{1}{2\pi\tau_P} = \frac{1}{2\pi(0.566\times10^{-9})} \Rightarrow 281\,\text{MHz}$$