Electronics II, Spring 2016 Exam3 Solution 2016/5/13

#1 (15%)

$$V_{BE1} + I_{REF}R_{E1} = V_{BE2} + I_0 R_{E2}$$

$$V_{BE1} - V_{BE2} = I_0 R_{E2} - I_{REF} R_{E1}$$

For matched transistors

$$V_{BE1} = V_T \ln\left(\frac{I_{REF}}{I_S}\right)$$

$$V_{BE2} = V_T \ln\left(\frac{I_0}{I_S}\right)$$

$$\text{Then } V_T \ln\left(\frac{I_{REF}}{I_0}\right) = I_0 R_{E2} - I_{REF} R_{E1}$$

#2 (20%)

$$I_{REF} = \left(\frac{50}{2}\right)(15)(V_{SG1} - 0.5)^2 = \left(\frac{50}{2}\right)(3)(V_{SG3} - 0.5)^2$$

$$V_{SG1} + V_{SG3} = 10 \Rightarrow V_{SG3} = 10 - V_{SG1}$$

$$\sqrt{\frac{15}{3}}(V_{SG1} - 0.5) = 10 - V_{SG1} - 0.5$$

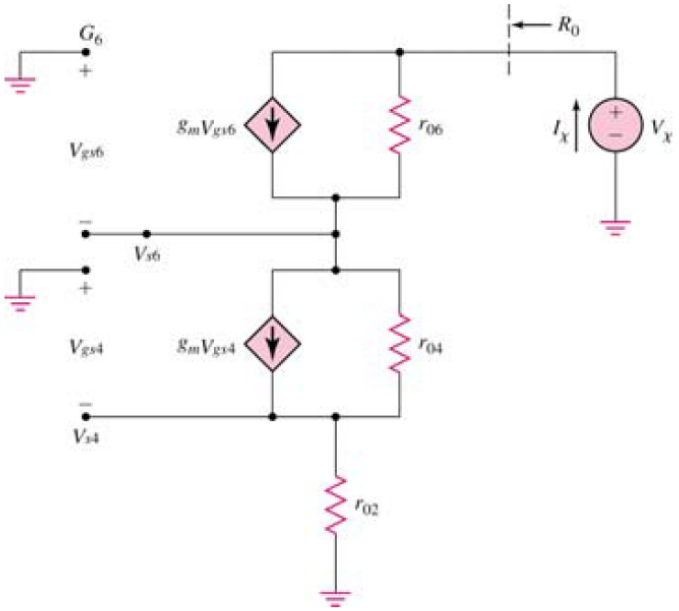
$$3.236 V_{SG1} = 10.618 \Rightarrow V_{SG1} = 3.28 \text{ V}$$

$$I_{REF} = \left(\frac{50}{2}\right)(15)(3.28 - 0.5)^2 \Rightarrow I_{REF} = 2.90 \text{ mA}$$

$$I_O = I_{REF} = 2.90 \text{ mA}$$

$$V_{SD2}(\text{sat}) = V_{SG2} + V_{TP} = 3.28 - 0.5 \Rightarrow \underline{V_{SD2}(\text{sat}) = 2.78 \text{ V}}$$

#3 (20%)



$$V_{gs4} = -I_X r_{02}$$

$$V_{S6} = \left(I_X - g_m V_{gs4}\right) r_{04} + I_X r_{02}$$

$$= \left(I_X + g_m I_X r_{02}\right) r_{04} + I_X r_{02}$$

$$V_{S6} = I_X \left[r_{02} + \left(1 + g_m r_{02}\right) r_{04} \right] = -V_{gs6}$$

$$I_X = g_m V_{gs6} + \frac{V_X - V_{S6}}{r_{06}} = \frac{V_X}{r_{06}} - V_{S6}\left(g_m + \frac{1}{r_{06}} \right)$$

$$I_X = \frac{V_X}{r_{06}} - I_X \left(g_m + \frac{1}{r_{06}} \right) \left[r_{02} + \left(1 + g_m r_{02}\right) r_{04} \right]$$

$$I_X \left\{ 1 + \left(g_m + \frac{1}{r_{06}} \right) \left[r_{02} + \left(1 + g_m r_{02}\right) r_{04} \right] \right\} = \frac{V_X}{r_{06}}$$

$$\frac{V_X}{I_X} = R_0 = r_{06} + \left(1 + g_m r_{06}\right) \left[r_{02} + \left(1 + g_m r_{02}\right) r_{04} \right]$$

$$I_0 \approx I_{REF} = 0.2 \text{ mA} = 0.2\left(V_{GS} - 1\right)^2$$

$$V_{GS} = 2 \text{ V}$$

$$g_m = 2K_n \left(V_{GS} - V_{TN}\right) = 2\left(0.2\right)\left(2 - 1\right) = 0.4 \ mA/V$$

$$r_{02} = r_{04} = r_{06} = \frac{1}{\lambda I_0} = \frac{1}{\left(0.02\right)\left(0.2\right)} = 250 \text{ k}\Omega$$

$$R_0 = 250 + \left[1 + \left(0.4\right)\left(250\right)\right] \times \left\{ 250 + \left[1 + \left(0.4\right)\left(250\right)\right]\left(250\right) \right\}$$

$$R_0 = 2575750 \text{ k}\Omega \Rightarrow \underline{R_0 = 2.58 \times 10^9 \ \Omega}$$

#4 (25%)

$$g_{m1} = 2\sqrt{\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_1 I_{D1}} = 2\sqrt{\left(\frac{0.1}{2}\right)(20)(0.1)} = 0.6325 \text{ mA/V}$$

$$r_{o1} = \frac{1}{\lambda_n I_{D1}} = \frac{1}{(0.02)(0.1)} = 500 \text{ k}\Omega$$

$$R_{o2} = r_{o2} + r_{o3}\left(1 + g_{m2} r_{o2}\right)$$

$$g_{m2} = 2\sqrt{\left(\frac{0.06}{2}\right)(80)(0.1)} = 0.9798 \text{ mA/V}$$

$$r_{o2} = r_{o3} = \frac{1}{(0.02)(0.1)} = 500 \text{ k}\Omega$$

$$R_{o2} = 500 + 500\left[1 + (0.9798)(500)\right] = 245{,}949 \text{ k}\Omega$$

$$A_\upsilon = -g_{m1}\left(r_{o1} \| R_{o2}\right) = -(0.6325)(500 \| 245{,}949) = -316$$

#5 (20%)

$$24 = I_{REF} R + V_{SG} + V_{GS}$$

$$V_{SG} = \sqrt{\frac{I_{REF}}{\left(\frac{k_p'}{2}\right)(1)}} - V_{TP} = 5.7735\sqrt{I_{REF}} + 0.8$$

$$V_{GS} = \sqrt{\frac{I_{REF}}{\left(\frac{k_n'}{2}\right)(1)}} + V_{TN} = 4.472\sqrt{I_{REF}} + 0.8$$

So $24 = I_{REF}(100) + 5.7735\sqrt{I_{REF}} + 0.8 + 4.4721\sqrt{I_{REF}} + 0.8$

Let $\sqrt{I_{REF}} = x$

Then $100x^2 + 10.2456x - 22.4 = 0 \Rightarrow x = 0.4248 \Rightarrow x^2 = I_{REF} = 0.1805 \text{ mA}$

$I_1 = (0.2) I_{REF} = 0.0361 \text{ mA}$

$I_2 = (1.25) I_{REF} = 0.2256 \text{ mA}$

$I_3 = (0.8) I_{REF} = 0.1444 \text{ mA}$

$I_4 = (4) I_{REF} = 0.722 \text{ mA}$