

#1

a.

$$I_1 = \frac{20 - V_{GS3}}{50} = 0.25(V_{GS3} - 2)^2$$

$$20 - V_{GS3} = 12.5(V_{GS3}^2 - 4V_{GS3} + 4)$$

$$12.5V_{GS3}^2 - 49V_{GS3} + 30 = 0$$

$$V_{GS3} = \frac{49 \pm \sqrt{(49)^2 - 4(12.5)(30)}}{2(12.5)} \Rightarrow \underline{V_{GS3} = 3.16 \text{ V}}$$

$$I_1 = \frac{20 - 3.16}{50} \Rightarrow \underline{I_1 = I_Q = 0.337 \text{ mA}}$$

$$I_{D1} = \frac{I_Q}{2} \Rightarrow \underline{I_{D1} = 0.168 \text{ mA}}$$

$$0.168 = 0.25(V_{GS1} - 2)^2 \Rightarrow V_{GS1} = 2.82 \text{ V}$$

$$V_{DS4} = -2.82 - (-10) \Rightarrow \underline{V_{DS4} = 7.18 \text{ V}}$$

$$V_{D1} = 10 - (0.168)(24) = 5.97 \text{ V}$$

$$V_{DS1} = 5.97 - (-2.82) \Rightarrow \underline{V_{DS1} = 8.79 \text{ V}}$$

(b)

$$\text{Max } v_{CM} \Rightarrow V_{DS1} = V_{DS2} = V_{DS}(\text{sat}) = V_{GS1} - V_{TN} \\ 2.82 - 2 = 0.82 \text{ V}$$

$$\text{Now } V_{D1} = 10 - (0.168)(24) = 5.97 \text{ V}$$

$$V_S(\text{max}) = 5.97 - V_{DS1}(\text{sat}) = 5.97 - 0.82$$

$$V_S(\text{max}) = 5.15 \text{ V}$$

$$v_{CM}(\text{max}) = V_S(\text{max}) + V_{GS1} = 5.15 + 2.82$$

$$\underline{v_{CM}(\text{max}) = 7.97 \text{ V}}$$

$$v_{CM}(\text{min}) = V^- + V_{DS4}(\text{sat}) + V_{GS1}$$

$$V_{DS4}(\text{sat}) = V_{GS4} - V_{TN} = 3.16 - 2 = 1.16 \text{ V}$$

$$\text{Then } v_{CM}(\text{min}) = -10 + 1.16 + 2.82 \Rightarrow \underline{v_{CM}(\text{min}) = -6.02 \text{ V}}$$

#2

See textbook for  $A_d$  and  $A_{cm}$  derivation.

a. For  $v_1 = v_2 = 0$  and neglecting base currents

$$R_E = \frac{-0.7 - (-10)}{0.15} \Rightarrow R_E = 62 \text{ k}\Omega$$

b.

$$A_d = \frac{v_{o2}}{v_d} = \frac{\beta R_C}{2(r_\pi + R_B)}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.075} = 34.7 \text{ k}\Omega$$

$$A_d = \frac{(100)(50)}{2(34.7 + 0.5)} \Rightarrow A_d = 71.0$$

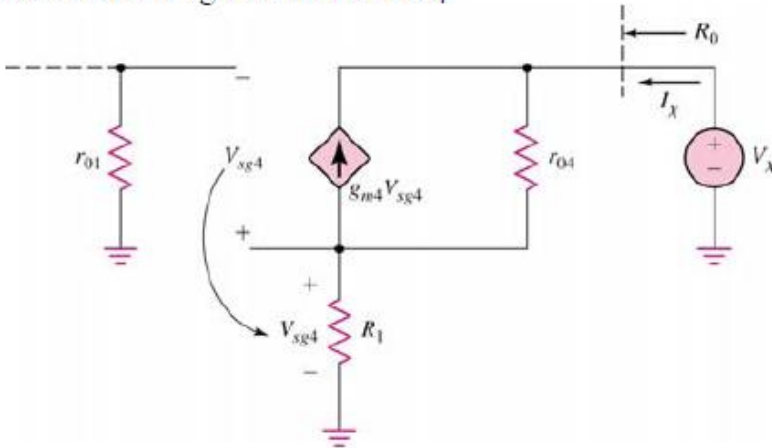
$$A_{cm} = -\frac{\beta R_C}{r_\pi + R_B} \left[ \frac{1}{1 + \frac{2R_E(1+\beta)}{r_\pi + R_B}} \right]$$

$$= -\frac{(100)(50)}{34.7 + 0.5} \left[ \frac{1}{1 + \frac{2(62)(101)}{34.7 + 0.5}} \right] \Rightarrow A_{cm} = -0.398$$

$$CMRR_{dB} = 20 \log_{10} \left| \frac{71.0}{0.398} \right| \Rightarrow CMRR_{dB} = 45.0 \text{ dB}$$

#3

Resistance looking into drain of  $M_4$ .



$$V_{sg4} \cong I_X R_1$$

$$I_X + g_{m4} V_{sg4} = \frac{V_X - V_{sg4}}{r_{o4}}$$

$$I_X \left[ 1 + g_{m4} R_1 + \frac{R_1}{r_{o4}} \right] = \frac{V_X}{r_{o4}}$$

$$\text{Or } R_o = r_{o4} \left[ 1 + g_{m4} R_1 + \frac{R_1}{r_{o4}} \right]$$

$$A_d = g_{m2} (r_{o2} \parallel R_o)$$

$$g_{m2} = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.080)(0.1)} \\ = 0.179 \text{ mA/V}$$

$$r_{o2} = \frac{1}{\lambda_n I_{DQ}} = \frac{1}{(0.015)(0.1)} = 667 \text{ k}\Omega$$

$$g_{m4} = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.080)(0.1)} \\ = 0.179 \text{ mA/V}$$

$$r_{o4} = \frac{1}{\lambda_p I_{DQ}} = \frac{1}{(0.02)(0.1)} = 500 \text{ k}\Omega$$

$$R_o = 500 \left[ 1 + (0.179)(1) + \frac{1}{500} \right] = 590.5 \text{ k}\Omega$$

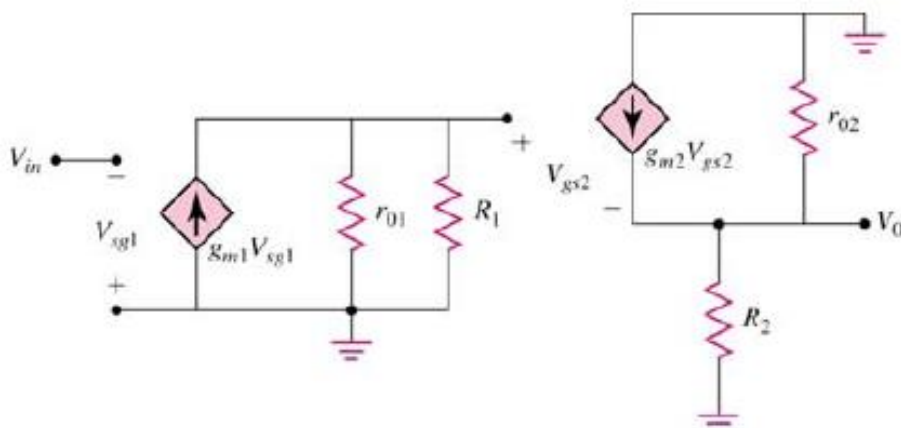
$$A_d = (0.179) [667 \parallel 590.5] \Rightarrow \underline{A_d = 56.06}$$

#4

$$I_2 = \frac{5}{5} = 1 \text{ mA}$$

$$V_{GS2} = \sqrt{\frac{1}{0.5}} + 0.8 = 2.21 \text{ V}$$

$$I_1 = \frac{2.21 - (-5)}{35} = 0.206 \text{ mA}$$



$$V_0 = (g_{m2} V_{gs2})(R_2 \parallel r_{02})$$

$$V_{gs2} = (g_{m1} V_{gs1})(r_{01} \parallel R_1) - V_0 \text{ and } V_{gs1} = -V_{in}$$

$$\text{So } V_{gs2} = -(g_{m1} V_{in})(r_{01} \parallel R_1) - V_0$$

Then

$$V_0 = g_{m2} (R_2 \parallel r_{02}) [-(g_{m1} V_{in})(r_{01} \parallel R_1) - V_0]$$

$$A_v = \frac{V_0}{V_{in}} = \frac{-g_{m2} (R_2 \parallel r_{02}) g_{m1} (r_{01} \parallel R_1)}{1 + g_{m2} (R_2 \parallel r_{02})}$$

$$g_{m2} = 2\sqrt{K_{n2} I_{D2}} = 2\sqrt{(0.5)(1)} = 1.414 \text{ mA/V}$$

$$g_{m1} = 2\sqrt{K_{p1} I_{D1}} = 2\sqrt{(0.2)(0.206)} = 0.406 \text{ mA/V}$$

$$r_{01} = \frac{1}{\lambda_1 I_{D1}} = \frac{1}{(0.01)(0.206)} = 485 \text{ k}\Omega$$

$$r_{02} = \frac{1}{\lambda_2 I_{D2}} = \frac{1}{(0.01)(1)} = 100 \text{ k}\Omega$$

$$R_2 \parallel r_{02} = 5 \parallel 100 = 4.76 \text{ k}\Omega$$

$$R_1 \parallel r_{01} = 35 \parallel 485 = 32.6 \text{ k}\Omega$$

$$\text{Then } A_v = \frac{-(1.414)(4.76)(0.406)(32.6)}{1 + (1.414)(4.76)}$$

$$\text{So } \Rightarrow \underline{A_v = -11.5}$$

$$\begin{aligned} R_0 &= \frac{1}{g_{m2}} \parallel R_2 \parallel r_{02} = \frac{1}{1.414} \parallel 5 \parallel 100 \\ &= 0.707 \parallel 4.76 \end{aligned}$$

$$\text{So } \underline{R_0 = 0.616 \text{ k}\Omega}$$