

1.

a.

$$I_0 = I_{C1} \text{ and } I_{REF} = I_{C1} + I_{B3} = I_{C1} + \frac{I_{E3}}{1 + \beta}$$

$$I_{E3} = I_{B1} + I_{B2} + \frac{V_{BE}}{R_2} = \frac{2I_{C1}}{\beta} + \frac{V_{BE}}{R_2}$$

$$I_{REF} = I_{C1} + \frac{2I_{C1}}{\beta(1 + \beta)} + \frac{V_{BE}}{(1 + \beta)R_2}$$

$$I_{REF} - \frac{V_{BE}}{(1 + \beta)R_2} = I_0 \left( 1 + \frac{2}{\beta(1 + \beta)} \right)$$

$$I_0 = \frac{I_{REF} - \frac{V_{BE}}{(1 + \beta)R_2}}{\left( 1 + \frac{2}{\beta(1 + \beta)} \right)}$$

$$I_{REF} = (0.70) \left( 1 + \frac{2}{(80)(81)} \right) + \frac{0.7}{(81)(10)}$$

$$I_{REF} = 0.700216 + 0.000864$$

$$\underline{I_{REF} = 0.7011 \text{ mA}} = \frac{10 - 2(0.7)}{R_1} \Rightarrow \underline{R_1 = 12.27 \text{ k}\Omega}$$

b.

2.

$$V_{BE1} = (0.026) \ln \left( \frac{100 \times 10^{-6}}{5 \times 10^{-15}} \right) = 0.61669 \text{ V}$$

$$V_{BE2} = V_{BE1} + I_{REF} R_E = 0.61669 + (0.1)(0.7) = 0.68669 \text{ V}$$

$$I_O = (5 \times 10^{-15}) \exp \left( \frac{0.68669}{0.026} \right) \Rightarrow I_O = 1.477 \text{ mA}$$

3.

$$I_{REF} = \frac{2.5 - V_{GS}}{15} = \left(\frac{0.08}{2}\right)(6)(V_{GS} - 0.5)^2$$

$$2.5 - V_{GS} = 3.6(V_{GS}^2 - V_{GS} + 0.25)$$

$$3.6V_{GS}^2 - 2.6V_{GS} - 1.6 = 0$$

$$V_{GS} = \frac{2.6 \pm \sqrt{6.76 + 23.04}}{2(3.6)}$$

$$V_{GS} = 1.12 \text{ V (1.1193)}$$

$$I_{REF} = \frac{2.5 - 1.1193}{15} \Rightarrow I_{REF} = 92.0 \mu\text{A (92.05)}$$

$$I_o = 92.0 \mu\text{A}$$


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4.

$$I_{REF} = \left(\frac{80}{2}\right)(25)(V_{SG1} - 1.2)^2 = \left(\frac{80}{2}\right)(4)(V_{SG3} - 1.2)^2$$

$$V_{SG1} + 2V_{SG3} = 10 \Rightarrow V_{SG3} = \frac{10 - V_{SG1}}{2}$$

$$\text{Then } \sqrt{\frac{25}{4}}(V_{SG1} - 1.2) = \frac{10 - V_{SG1}}{2} - 1.2$$

$$3V_{SG1} = 6.8 \Rightarrow V_{SG1} = 2.27 \text{ V}$$

$$I_{REF} = \left(\frac{80}{2}\right)(25)(2.267 - 1.2)^2 \Rightarrow \underline{I_{REF} = I_o = 1.14 \text{ mA}}$$


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5.

$$I_{REF} = K_n (V_{GS} - V_{TN})^2$$

$$0.020 = 0.080(V_{GS} - 1)^2$$

$$V_{GS} = 1.5 \text{ V all transistors}$$

$$R_o = r_{o4} + r_{o2}(1 + g_m r_{o4})$$

$$r_{o2} = r_{o4} = \frac{1}{\lambda I_o} = \frac{1}{(0.02)(0.020)} = 2500 \text{ k}\Omega$$

$$g_m = 2K_n (V_{GS} - V_{TN}) = 2(0.080)(1.5 - 1) \Rightarrow g_m = 0.080 \text{ mA/V}$$

$$R_o = 2500 + 2500(1 + (0.080)(2500)) \Rightarrow \underline{R_o = 505 \text{ M}\Omega}$$


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6.

$$I_{D2} = \frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1} \cdot I_{REF} = \frac{9}{15}(200) \Rightarrow \underline{I_{D2} = 120 \mu\text{A}}$$

$$I_O = \frac{\left(\frac{W}{L}\right)_4}{\left(\frac{W}{L}\right)_3} \cdot I_{D2} = \left(\frac{20}{9}\right)(120) \Rightarrow \underline{I_O = 267 \mu\text{A}}$$

$$I_O = 266.7 = \left(\frac{40}{2}\right)(20)(V_{SG4} - 0.6)^2$$

$$V_{SG4} = 1.416 \text{ V}$$

$$V_{SD4}(\text{sat}) = 1.416 - 0.6 \Rightarrow \underline{V_{SD4}(\text{sat}) = 0.816 \text{ V}}$$

7.

**Solution:** Since  $M_1$  and  $M_2$  are matched, then  $I_O = I_{REF}$ , and the transconductance is

$$g_m = 2\sqrt{K_n I_{REF}} = 2\sqrt{(1)(0.5)} = 1.41 \text{ mA/V}$$

The small-signal transistor conductances are

$$g_o = g_{o2} = \lambda I_{REF} = (0.01)(0.5) = 0.005 \text{ mA/V}$$

For  $R_L = 100 \text{ k}\Omega$  ( $g_L = 0.01 \text{ mA/V}$ ), the voltage gain is

$$A_v = \frac{-g_m}{g_o + g_L + g_{o2}} = \frac{-1.41}{0.005 + 0.01 + 0.005} = -70.5$$