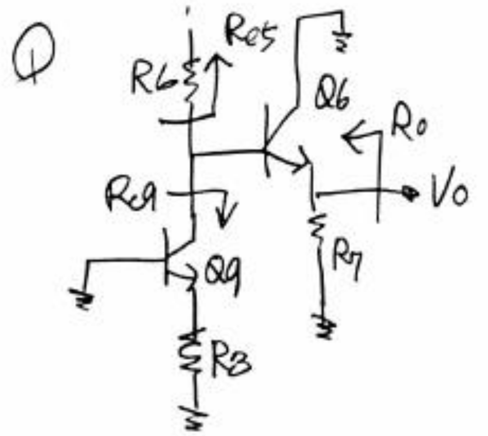
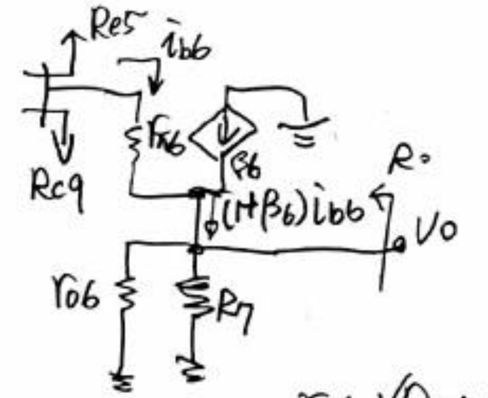
Electronics II 2021.
25th June, D.C. Chang

#1

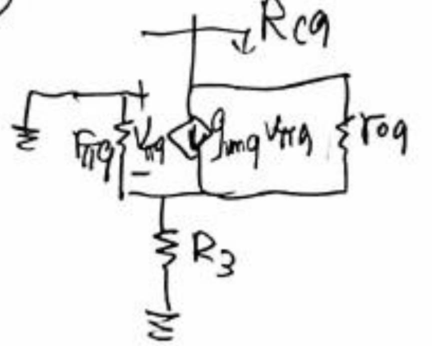
①





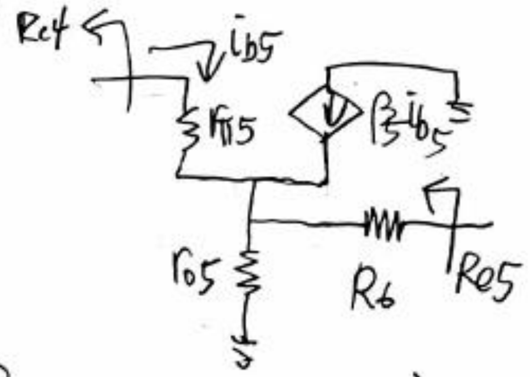
$$R_o = r_{o6} // R_7 // \frac{r_{\pi 6} + (R_{e5} // R_{c9})}{1+\beta_6}$$

②



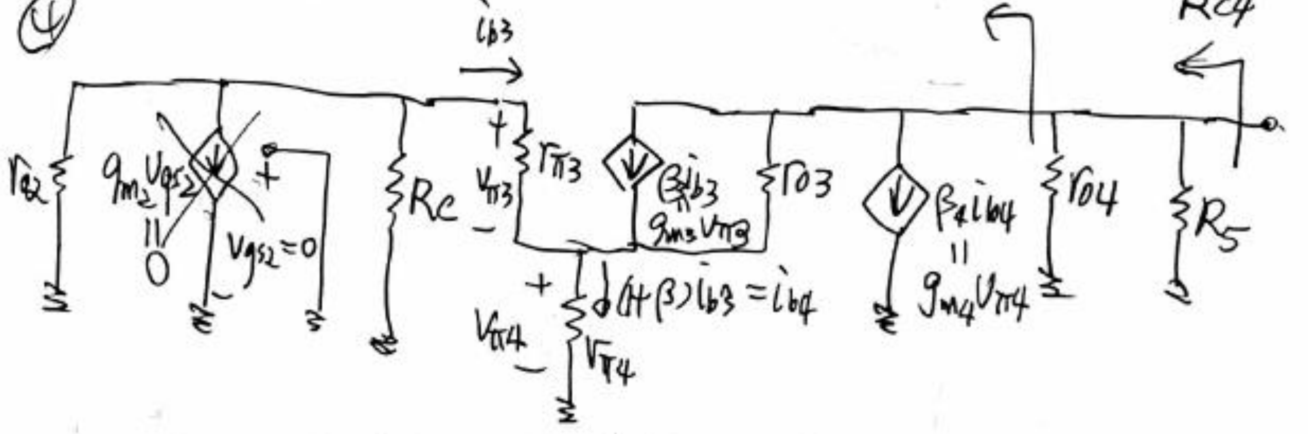
$$R_{c9} = (r_{\pi 9} // R_3) + r_{o9}\left(1 + g_{m9}(r_{\pi 9} // R_3)\right)$$

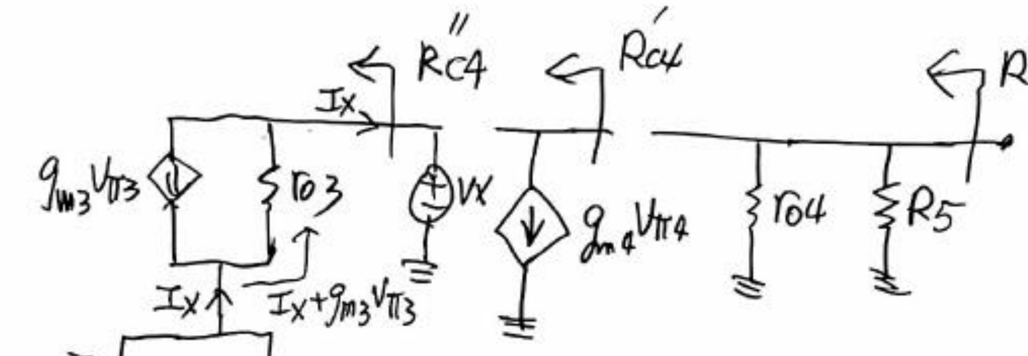
③

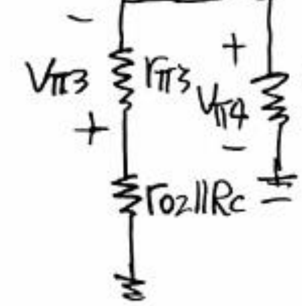


$$R_{e5} = R_6 + r_{o5} // \frac{r_{\pi 5} + R_{c4}}{1+\beta_5}$$

④







$$V_{\pi 3} = \frac{r_{\pi 4} \cdot I_x \cdot r_{\pi 3}}{r_{\pi 3} + r_{o2}//R_C + r_{\pi 4}}$$

$$V_x = -\left[r_{\pi 4}//(r_{\pi 3} + r_{o2}//R_C)\right] I_x - \left(I_x + \frac{r_{\pi 4} I_x \cdot r_{\pi 3}\, g_{m3}}{r_{\pi 3} + r_{o2}//R_C + r_{\pi 4}}\right) r_{o3}$$

$$R''_{c4} = -\frac{V_x}{I_x} = r_{\pi 4}//(r_{\pi 3} + r_{o2}//R_C) + \left(1 + \frac{\beta_3\, r_{\pi 4}}{r_{\pi 3} + r_{o2}//R_C + r_{\pi 4}}\right) r_{o3}$$

$$\approx \frac{\beta_3\, r_{\pi 4} \cdot r_{o3}}{r_{\pi 3} + r_{\pi 4} + r_{o2}//R_C} \approx \beta_3 r_{o3} \quad \text{(order of magnitude)}$$

$$V_{\pi 4} = -\frac{r_{\pi 3} + r_{o2}//R_C}{r_{\pi 3} + r_{\pi 4} + r_{o2}//R_C} I_x \cdot r_{\pi 4}$$

$$R'_{c4} = \frac{V_x}{g_{m4} V_{\pi 4}} \cong \frac{\dfrac{\beta_3\, r_{\pi 4} \cdot r_{o3}}{r_{\pi 3} + r_{\pi 4} + r_{o2}//R_C} \cdot I_x}{\dfrac{\beta_4 (r_{\pi 3} + r_{o2}//R_C)}{r_{\pi 3} + r_{\pi 4} + r_{o2}//R_C} \cdot I_x}$$

$$\approx \frac{\beta_3 \cdot r_{\pi 4} \cdot r_{o3}}{\beta_4 (r_{\pi 3} + r_{o2}//R_C)} \approx r_{o3} \quad \text{(order of magnitude)}$$

$$R_{c4} = R_5 // r_{o4} // R'_{c4} // R''_{c4} \quad \divideontimes$$

* Approximation :

$$R_{c4} \approx R_5$$

$$R_{e5} \simeq R_6 + \frac{r_{\pi 5} + R_5}{1+\beta_5}$$

$$R_{c9} \approx g_{m9} (r_{\pi 9} // R_3) \cdot r_{o9}$$

$$R_o \approx \frac{r_{\pi 6} + R_{e5}}{1+\beta_6} // R_7$$

$$\approx \frac{1}{1+\beta_6}\left(r_{\pi 6} + R_6 + \frac{r_{\pi 5} + R_5}{1+\beta_5}\right) // R_7 \quad \divideontimes$$

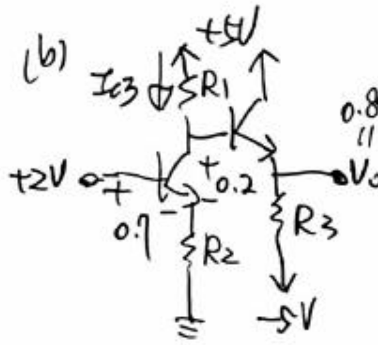
#2. (a) $V_{o2} = 2\,(V)$, $I_R = \frac{I_Q}{2} = 0.25\,(mA)$

$$R = \frac{5-2}{0.25} = 12\,(k\Omega)$$

$$V_2 = V_{o2} - V_{DS2} + V_{GS2} \leq V_{o2} - V_{DS2}(sat) + V_{GS2}$$

$$= V_{o2} - (V_{GS2} - V_{TN}) + V_{GS2} = V_{o2} + V_{TN} = 2.8\,(V)$$

(b)



$$V_o = 2 - 0.7 + V_{CE3} - 0.7 \geq 0.6 + V_{CE3}(sat)$$

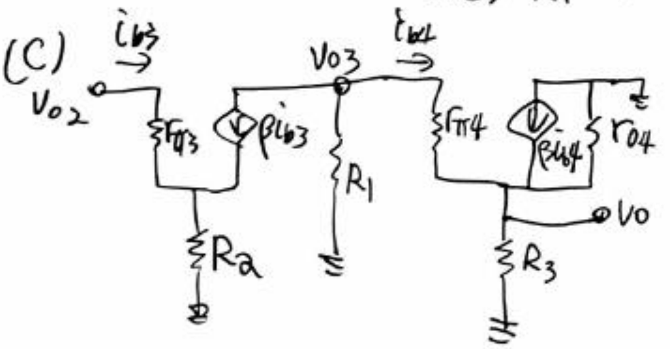
$$V_{CE3}(sat) = 0.2\,(V)$$

$$\therefore V_o \geq 0.6 + 0.2 = 0.8\,(V),\quad V_o(min) = 0.8\,(V)$$

$$I_{C3} \cdot R_2 = 2 - 0.7 = 1.3,\quad R_2 = \frac{1.3}{0.25} = 5.2\,(k\Omega)$$

$$I_{C3} \cdot R_1 = 5 - (0.8 + 0.7) = 3.5,\quad R_1 = \frac{3.5}{0.25} = 14\,(k\Omega)$$

(C)



$$V_{o2} = i_{b3} \cdot [r_{\pi3} + (1+\beta) R_2]$$

$$V_{o3} = -\beta i_{b3} [R_1 // (r_{\pi4} + (1+\beta)(R_3 // r_{o4}))]$$

$$V_{o3} = i_{b4} [r_{\pi4} + (1+\beta)(R_3 // r_{o4})]$$

$$V_o = (1+\beta) i_{b4} \cdot (R_3 // r_{o4})$$

$$\frac{V_o}{V_{o2}} = \frac{V_{o3}}{V_{o2}} \cdot \frac{V_o}{V_{o3}}$$

$$= \frac{-\beta [R_1 // (r_{\pi4} + (1+\beta)(R_3 // r_{o4}))]}{r_{\pi3} + (1+\beta) R_2} \times \frac{(1+\beta)(R_3 // r_{o4})}{r_{\pi4} + (1+\beta)(R_3 // r_{o4})}$$

$$r_{\pi3} = \frac{\beta V_T}{I_{C3}} = \frac{250 \times 0.026}{0.25} = 26\,(k\Omega)$$

$$r_{\pi4} = \frac{\beta V_T}{I_{C4}} = \frac{250 \times 0.026}{2} = 3.25\,(k\Omega)$$

$$r_{o4} = \frac{V_{A4}}{I_{C4}} = \frac{90}{2} = 45\,(k\Omega)$$

$$R_3 = \frac{0.8 - (-5)}{2} = 2.9\,(k\Omega)$$

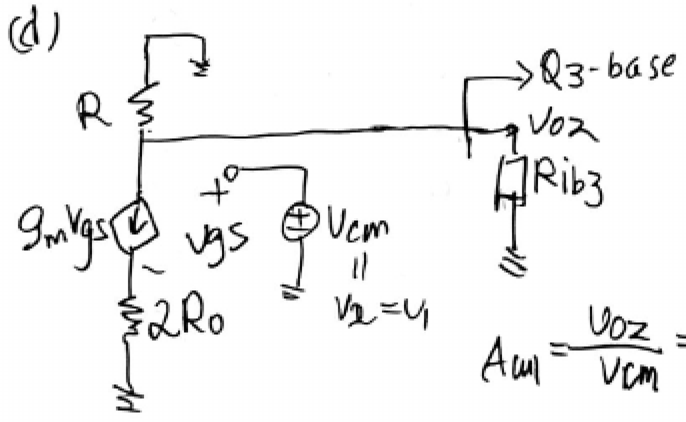
$$R_3 // r_{o4} = 2.9 // 45 = 2.7244$$

$$r_{\pi4} + (1+\beta)(R_3 // r_{o4}) = 3.25 + 251 \times 2.7244 = 687.0744$$

$$\frac{V_o}{V_{o2}} = \frac{-250 \times (14 // 687)}{26 + 251 \times 5.2} \times \frac{251 \times 2.7244}{687.0744} = -\frac{3430}{1331.2} \times 0.995$$

$$= -2.564$$

(d)



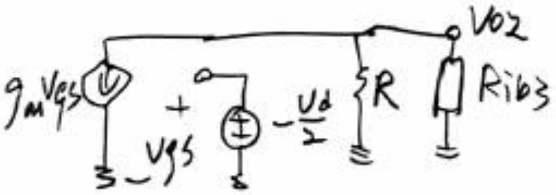
$$v_{cm} = V_{gs} + g_m V_{gs} \cdot 2R_o$$

$$V_{gs} = \frac{V_{cm}}{1+2g_m R_o}$$

$$V_{o2} = -g_m V_{gs}\left[R \,//\, (r_{\pi 3}+(1+\beta)R_2)\right]$$

$$A_{cm} = \frac{V_{o2}}{V_{cm}} = \frac{-g_m\left[R\,//\,(r_{\pi 3}+(1+\beta)R_2)\right]}{1+2g_m R_o}$$

(e) $A_d = \frac{V_{o2}}{V_d}$



$$v_{o2} = \frac{1}{2} g_m V_d \cdot (R\,//\,R_{ib3})$$

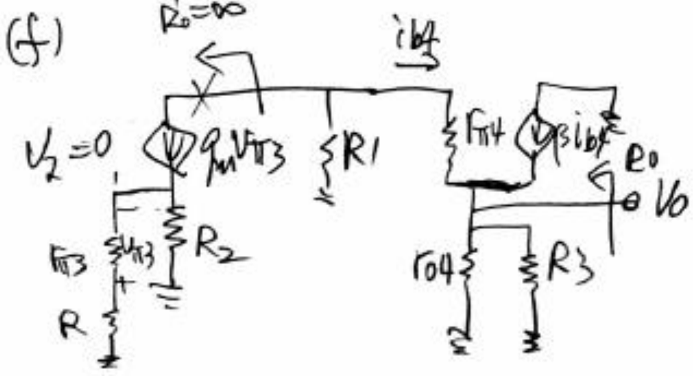
$$A_d = \frac{1}{2} g_m (R\,//\,R_{ib3})$$

$$CMRR = \left|\frac{A_d}{A_{cm}}\right| = \left|\frac{\frac{1}{2}g_m}{\frac{g_m}{1+2g_m R_o}}\right| = \frac{1+2g_m R_o}{2}$$

$$g_m = \sqrt{2K_n I_Q} = \sqrt{2\times 0.2\times 0.5} = 0.4472 \text{ (mA/V)}$$

$$CMRR_{dB} = 20\log_{10}\left(\frac{1+2\times 0.4472\times 200}{2}\right) = 20\log_{10}(89.94) = 39.079 \text{ (dB)}$$

(f)

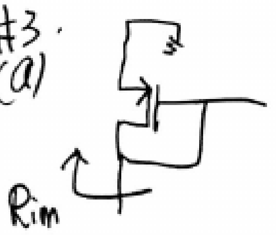


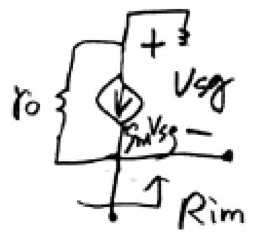
$$R_o = r_{o4}\,//\,R_3\,//\,\frac{r_{\pi 4}+R_1}{1+\beta}$$

$$R_o = 45\,//\,2.9\,//\,\frac{3.25+14}{251} = 45\,//\,2.9\,//\,0.0687 = 0.067 \text{ (k}\Omega\text{)}$$

#3.

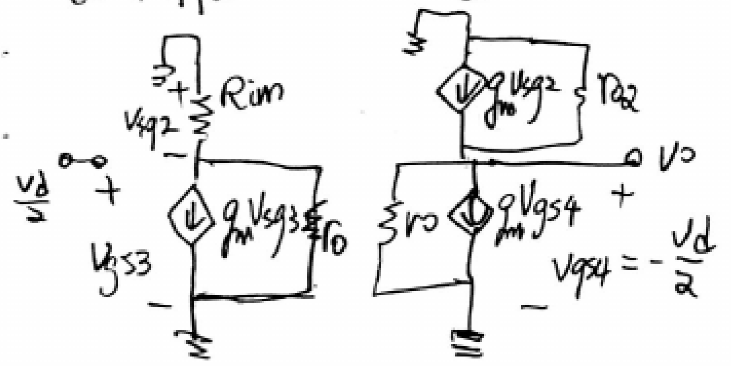
(a)





$$R_{im} = r_o \parallel \frac{1}{g_m}$$

(b) Differential-mode



$$R_o = r_o$$

$$G_{V_2} = -\frac{1}{2} g_m v_d$$

$$R_{om} = r_{o2} = r_o$$

$$V_{sg2} = i_d \cdot R_{im}$$

$$= \frac{1}{2} g_m v_d \left(\frac{1}{g_m} \parallel r_o\right)$$

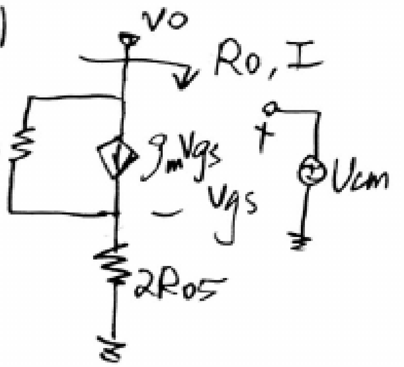
$$\cong \frac{1}{2} v_d$$

$$V_o = \left(\frac{1}{2} g_m v_d\right) \times 2 \times (r_o \parallel r_o)$$

$$= g_m v_d \cdot \frac{r_o}{2} = \frac{1}{2} g_m r_o v_d$$

$$A_d = \frac{v_o}{v_d} = \frac{1}{2} g_m r_o$$

(c)



① R_o : ($V_{cm} = 0$)

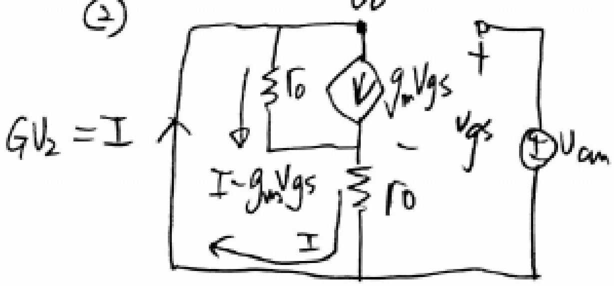
$$R_{no} = 2R_{o5} + r_o(1 + 2g_m R_{o5})$$

$$= 2R_{o5} + r_o + 2g_m r_o R_{o5}$$

$$R_{o5} = \frac{r_o}{2}$$

$$\therefore R_o = 2r_o + g_m r_o^2 \approx g_m r_o^2$$

②



$$V_{cm} = V_{gs} + I \cdot r_o, \quad V_{gs} = V_{cm} - I r_o$$

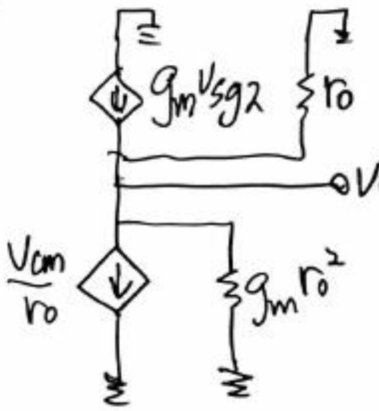
$$(I - g_m V_{gs}) r_o + I r_o = 0$$

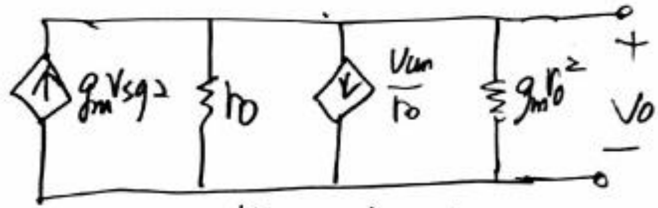
$$I r_o - g_m r_o (V_{cm} - I r_o) + I r_o = 0$$

$$2 I r_o + I \cdot g_m r_o^2 = g_m r_o V_{cm}$$

$$G_{V_2} = I = \frac{g_m V_{cm}}{2 + g_m r_o} \approx \frac{V_{cm}}{r_o}$$

(d)





$$v_{sg2} = i_d \cdot \left(\frac{1}{g_m} \| r_o\right) = \frac{v_{cm}}{r_o} \cdot \left(\frac{1}{g_m} \| r_o\right)$$

$$v_o = -\frac{v_{cm}}{r_o}\left(1 - g_m\left(\frac{1}{g_m} \| r_o\right)\right) \cdot \left(r_o \| g_m r_o^2\right)$$

$$\approx -\frac{v_{cm}}{g_m r_o^2} \cdot \left(r_o \| g_m r_o^2\right)$$

$$A_{cm} = \frac{v_o}{v_{cm}} \approx -\frac{r_o \| g_m r_o^2}{g_m r_o^2} \approx \frac{-1}{g_m r_o}$$

(e)

$$CMRR = \left|\frac{A_d}{A_{cm}}\right| = \frac{\frac{1}{2} g_m r_o}{\frac{1}{g_m r_o}} = \frac{1}{2} g_m^2 r_o^2$$

$$g_m = \sqrt{2 k_n I_Q}, \quad r_o = \frac{2}{\lambda I_Q}$$

$$CMRR = \frac{1}{2} \cdot 2 k_n I_Q \cdot \frac{4}{\lambda^2 I_Q^2} = \frac{4 K_n}{\lambda^2 I_Q} \quad \#$$