Electronics II Midterm Solution   2024/3/29

(1)

(a)

$$V_G = \left(\frac{R_2}{R_1+R_2}\right)V_{DD} \quad = \left(\frac{166}{166+234}\right)(10)$$

$$= 4.15 \text{ V}$$

$$I_D = \frac{V_G - V_{GS}}{R_S} = K_n\left(V_{GS} - V_{TN}\right)^2$$

$$4.15 - V_{GS} = (0.5)(0.5)\left(V_{GS}^2 - 4V_{GS} + 4\right)$$

$$0.25V_{GS}^2 - 3.15 = 0 \Rightarrow V_{GS} = 3.55 \text{ V}$$

$$g_m = 2K_n\left(V_{GS} - V_{TN}\right) = 2(0.5)(3.55-2)$$

$$g_m = 1.55 \ mA/V$$

$$R_0 = R_S \left\| \frac{1}{g_m} = 0.5 \right\| \frac{1}{1.55} = 0.5 \| 0.645$$

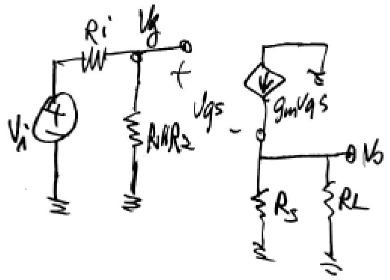
$$R_0 = 0.282 \text{ k}\Omega$$

$$r = \left(R_0 \| R_L\right)C_L \text{ and } f_H = \frac{1}{2\pi r}$$

$$\beta\omega \approx f_H = 5 \text{ MHz} \Rightarrow r = \frac{1}{2\pi\left(5\times10^6\right)} = 3.18\times10^{-8} \text{ s}$$

$$C_L = \frac{r}{R_0 \| R_L} = \frac{3.18\times10^{-8}}{(0.282\|4)\times10^3} \Rightarrow \underline{C_L = 121 \text{ pF}}$$

(b)



$$V_g = \frac{R_1 /\!/ R_2}{R_1 /\!/ R_2 + R_i} \cdot V_i$$

$$V_g = V_{gs} + g_m (R_S /\!/ R_L) V_{gs}$$
$$= [1 + g_m (R_S /\!/ R_L)] \cdot V_{gs}$$

$$V_o = g_m (R_S /\!/ R_L) V_{gs}$$
$$= g_m (R_S /\!/ R_L) \cdot \frac{V_g}{1 + g_m (R_S /\!/ R_L)}$$

$$\Rightarrow A_v = \frac{V_o}{V_i} = \frac{g_m (R_S /\!/ R_L)}{1 + g_m (R_S /\!/ R_L)} \cdot \frac{R_1 /\!/ R_2}{R_1 /\!/ R_2 + R_i}$$

$$g_m = 1.55 \text{ mA/V}$$
$$R_S /\!/ R_L = \frac{2}{4.5} = 0.445 \text{ K}\Omega$$
$$R_1 /\!/ R_2 = 97.11 \text{ K}\Omega$$
$$A_v = \frac{1.55 \times 0.445}{1 + 1.55 \times 0.445} \times \frac{97.11}{97.11 + 2} = 0.4$$

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(2)

(a) $$\frac{V_{gs}}{V_i} = \frac{-\left(\frac{1}{g_m} \Big\| \frac{1}{sC_i}\right)}{\left(\frac{1}{g_m} \Big\| \frac{1}{sC_i}\right) + R_S}$$

Now $\left(\frac{1}{g_m}\middle\|\frac{1}{sC_i}\right)=\frac{\left(\frac{1}{g_m}\right)\left(\frac{1}{sC_i}\right)}{\frac{1}{g_m}+\frac{1}{sC_i}}=\frac{\frac{1}{g_m}}{1+s\left(\frac{1}{g_m}\right)C_i}$

So $\frac{V_{gs}}{V_i}=\frac{-\frac{1}{g_m}}{\frac{1}{g_m}+R_S\left(1+s\left(\frac{1}{g_m}\right)C_i\right)}=\left(\frac{-\frac{1}{g_m}}{\frac{1}{g_m}+R_S}\right)\cdot\frac{1}{\left[1+s\left(\frac{1}{g_m}\middle\|R_S\right)C_i\right]}$

We have

$$V_o=-g_mV_{gs}\left[\frac{R_D}{R_D+R_L+\frac{1}{sC_C}}\right]\cdot R_L=-g_mV_{gs}\left[\frac{R_DR_L(sC_C)}{1+s(R_D+R_L)C_C}\right]$$

$$V_o=-g_mV_{gs}\left(\frac{R_DR_L}{R_D+R_L}\right)\left[\frac{s(R_D+R_L)C_C}{1+s(R_D+R_L)C_C}\right]$$

Then $T(s)=\frac{V_o(s)}{V_i(s)}=\frac{+g_m(R_D\|R_L)}{1+g_mR_S}\cdot\frac{1}{\left[1+s\left(\frac{1}{g_m}\middle\|R_S\right)C_i\right]}\cdot\left[\frac{s(R_D+R_L)C_C}{1+s(R_D+R_L)C_C}\right]$

$$\tau=\left(\frac{1}{g_m}\middle\|R_S\right)C_i$$

(b)

$$\tau=(R_D+R_L)C_C$$

Notice: You need to explain why the output resistance looking into the drain is **infinity**.

We have explained this several times in class. Please refer to our class video.

The score is 0 if the answer is given without definitely explaining the reason!!

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(3)

$$R_{TH} = R_1 \| R_2 = 60 \| 5.5 = 5.04 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) V_{CC} = \left(\frac{5.5}{5.5 + 60} \right) (15) = 1.26 \text{ V}$$

$$I_{BQ} = \frac{1.26 - 0.7}{5.04 + (101)(0.2)} = 0.0222 \text{ mA}$$

$$I_{CQ} = 2.22 \text{ mA}$$

$$r_\pi = \frac{(100)(0.026)}{2.22} = 1.17 \text{ k}\Omega$$

$$g_m = \frac{2.22}{0.026} = 85.4 \text{ mA/V}$$

Lower 3 – *dB* frequency:

$$\tau_L = R_{eq} \cdot C_{C1}$$

$$R_{eq} = R_S + R_1 \| R_2 \| r_\pi$$

$$= 2 + 60 \| 5.5 \| 1.17 = 2.95 \text{ k}\,\Omega$$

$$\tau_L = (2.95 \times 10^3)(0.1 \times 10^{-6}) = 2.95 \times 10^{-4} \text{ s}$$

$$f_L = \frac{1}{2\pi \tau_L} = \frac{1}{2\pi (2.95 \times 10^{-4})} \Rightarrow f_L = 540 \text{ Hz}$$

Upper 3 – *dB* frequency:

$$f_T = \frac{g_m}{2\pi (C_\pi + C_\mu)} \Rightarrow 400 \times 10^6 = \frac{85.4 \times 10^{-3}}{2\pi (C_\pi + C_\mu)}$$

$$C_\pi + C_\mu = 34 \text{ pF}; \quad C_\mu = 2 \text{ pF}; \quad C_\pi = 32 \text{ pF}$$

$$C_M = C_\mu (1 + g_m R_C) = 2[1 + (85.4)(4)] \Rightarrow C_M = 685 \text{ pF}$$

$$C_i = C_\pi + C_M = 32 + 685 = 717 \text{ pF}$$

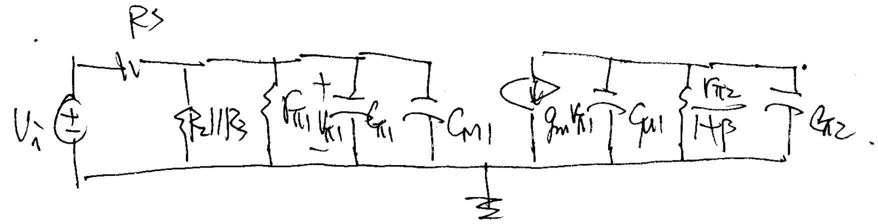
$$R_{eq} = R_S \| R_1 \| R_2 \| r_\pi = 2 \| 60 \| 5.5 \| 1.17 \Rightarrow R_{eq} = 0.644 \text{ k}\,\Omega$$

$$\tau = R_{eq} \cdot C_i = (0.644 \times 10^3)(717 \times 10^{-12})$$

$$= 4.62 \times 10^{-7} \text{ s}$$

$$f_H = \frac{1}{2\pi \tau} \Rightarrow f_H = 344 \text{ kHz}$$

(4)

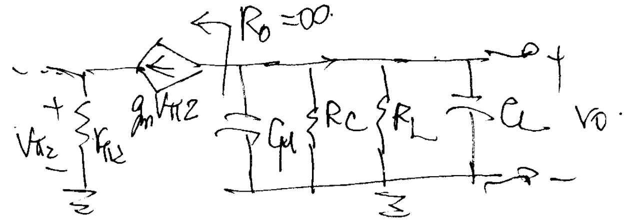


(a). $C_{M1} = C_{\mu1}\left(1+\frac{g_{m2} r_{\pi2}}{1+\beta}\right) = 2C_{\mu1}$.

$\tau_{\pi1} = (R_S // R_2 // R_3 // r_{\pi1}) \cdot (C_{\pi1} + C_{M1})$

$$f_{H\pi} = \frac{1}{2\pi(R_S//R_2//R_3//r_{\pi1})(C_{\pi1}+2C_{\mu1})} \quad ※$$

$r_\pi = \frac{I_{CQ}}{V_T}$, $C_{\pi1} = C_\pi$, $C_{\mu1} = C_\mu$.



$\tau_M = (R_C // R_L)(C_M + C_L)$

$$f_{HM} = \frac{1}{2\pi\tau_M} = \frac{1}{2\pi(R_C//R_L)(C_M+C_L)} \quad ※$$

(b) $V_o = -g_m(R_C // R_L) \cdot V_{\pi2}$

$= -g_m(R_C//R_L) \cdot \frac{g_{m1} r_{\pi2}}{1+\beta} V_{\pi1}$

$= \underbrace{-g_m(R_C//R_L) \cdot \frac{R_2//R_3//r_\pi}{R_2//R_3//r_\pi + R_S}}_{A_V} V_i$.

$r_\pi = 120 \times \frac{0.026}{1.02}$
$= 3K$.

(c) ① $C_L \to 0$

$$f_{H\pi} = \frac{10^9}{2\pi \cdot (0.1 // 20.5 // 28.3 // 3)(12+4)} = \frac{10^9}{2 \times 3.14 \times 0.096 \times 16} = 103.67 MHz.$$

$$f_{HM} = \frac{10^9}{2\pi \times (5//10) \times 2} = \frac{10^9}{2 \times 3.14 \times 3.33 \times 2} = \underline{23.9 MHz} \text{ (dominant)}$$

② $C_L = 15pF$

$$f_{HM} = \frac{10^9}{2\pi \times 3.33 \times 17} = \underline{2.8 MHz} \text{ (dominant)}.$$