

1.

$$a. \quad I_1 = \frac{24 - V_{GS4}}{R_1} = k_n(V_{GS4} - V_{Th})^2$$

$$24 - V_{GS4} = (55)(0.2)(V_{GS4} - 2)^2$$

$$24 - V_{GS4} = 11(V_{GS4}^2 - 4V_{GS4} + 4)$$

$$11V_{GS4}^2 - 43V_{GS4} + 20 = 0$$

$$V_{GS4} = \frac{43 \pm \sqrt{(43)^2 - 4(11)(20)}}{2(11)} = 3.37 \text{ V}$$

Ideal, $A_{cm} = 0$

$$v_{o3} = A_d v_d = (-3.73)(0.3)$$

Or

$$\Rightarrow \underline{v_{o3} = -1.12 \sin \omega t \text{ (V)}}$$

$$I_1 = \frac{24 - 3.37}{55} = 0.375 \text{ mA} = I_Q$$

$$v_{o2} = 12 - \left(\frac{0.375}{2}\right)(40) = 4.5 \text{ V}$$

$$\frac{v_{o2} - V_{GS3}}{R_5} = I_{D3} = k_n(V_{GS3} - V_{Th})^2$$

$$4.5 - V_{GS3} = (0.2)(6)(V_{GS3}^2 - 4V_{GS3} + 4)$$

$$1.2V_{GS3}^2 - 3.8V_{GS3} + 0.3 = 0$$

$$V_{GS3} = \frac{3.8 \pm \sqrt{(3.8)^2 - 4(1.2)(0.3)}}{2(1.2)} = 3.09 \text{ V}$$

$$I_{D3} = \frac{4.5 - 3.09}{6} = 0.235 \text{ mA}$$

$$g_{m2} = 2\sqrt{K_n I_{D2}} = 2\sqrt{(0.2)\left(\frac{0.375}{2}\right)}$$

$$= 0.387 \text{ mA/V}$$

$$A_{d1} = \frac{1}{2}g_{m2}R_D = \frac{1}{2}(0.387)(40)$$

$$\Rightarrow A_{d1} = 7.74$$

$$A_2 = \frac{-g_{m3}R_{D2}}{1 + g_{m3}R_5}$$

$$g_{m3} = 2\sqrt{K_n I_{D3}} = 2\sqrt{(0.2)(0.235)}$$

$$= 0.434 \text{ mA/V}$$

$$A_2 = \frac{-(0.434)(4)}{1 + (0.434)(6)} = -0.482$$

$$\text{So } A_d = A_{d1} \cdot A_2 = (7.74)(-0.482)$$

$$\Rightarrow \underline{A_d = -3.73}$$

$$R_o = r_{o3} = \frac{1}{\lambda I_Q} = \frac{1}{(0.02)(0.375)} = 133 \text{ k}\Omega$$

$$A_{cm1} = \frac{-g_{m2}R_D}{1 + 2g_{m2}R_o} = \frac{-(0.387)(40)}{1 + 2(0.387)(133)}$$

$$= -0.149$$

$$A_{cm} = (-0.149)(-0.482) \Rightarrow \underline{A_{cm} = 0.0718}$$

$$b. \quad v_d = v_1 - v_2 = 0.3 \sin \omega t$$

$$v_{cm} = \frac{v_1 + v_2}{2} = 2 \sin \omega t$$

$$v_{o3} = A_d v_d + A_{cm} v_{cm}$$

$$= (-3.73)(0.3) + (0.0718)(2)$$

$$\Rightarrow \underline{v_{o3} = -0.975 \sin \omega t \text{ (V)}}$$

2.

$$R_i = r_{\pi 1} + (1 + \beta)r_{\pi 2}$$

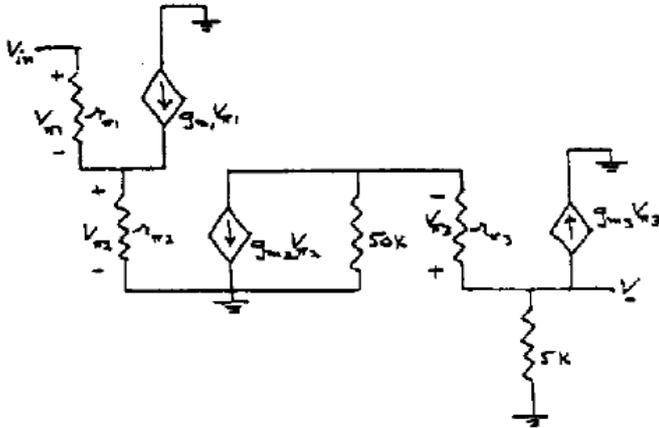
$$r_{\pi 2} = \frac{(100)(0.026)}{0.5} = 5.2 \text{ k}\Omega$$

$$r_{\pi 1} = \frac{(100)(0.026)}{(0.5/100)} = \frac{(100)^2(0.026)}{0.5} = 520 \text{ k}\Omega$$

$$R_i = 520 + (101)(5.2) \Rightarrow \underline{R_i \approx 1.05 \text{ M}\Omega}$$

$$R_0 = 5 \parallel \frac{r_{\pi 3} + 50}{101}, \quad r_{\pi 3} = \frac{(100)(0.026)}{1} = 2.6 \text{ k}\Omega$$

$$R_0 = 5 \parallel \frac{2.6 + 50}{101} = 5 \parallel 0.521 \Rightarrow \underline{R_0 = 0.472 \text{ k}\Omega}$$



$$V_0 = -\left(\frac{V_{\pi 3}}{r_{\pi 3}} + g_{m3} V_{\pi 3}\right)(5)$$

$$V_0 = -V_{\pi 3} \left(\frac{1 + \beta}{r_{\pi 3}}\right)(5) \quad (1)$$

$$\frac{V_{\pi 3}}{r_{\pi 3}} = g_{m2} V_{\pi 2} + \frac{(V_0 - V_{\pi 3})}{50}$$

$$g_{m2} V_{\pi 2} = V_{\pi 3} \left(\frac{1}{r_{\pi 3}} + \frac{1}{50}\right) - \frac{V_0}{50} \quad (2)$$

$$V_{\pi 2} = \left(\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1} V_{\pi 1}\right) r_{\pi 2} \quad (3)$$

$$= V_{\pi 1} \left(\frac{1 + \beta}{r_{\pi 1}}\right) r_{\pi 2}$$

and

$$V_{in} = V_{\pi 1} + V_{\pi 2} \quad (4)$$

$$g_{m2} = \frac{0.5}{0.026} = 19.23 \text{ mA/V}$$

Then

$$V_0 = -V_{\pi 3} \left(\frac{101}{2.6}\right)(5) \quad (1)$$

$$\Rightarrow V_{\pi 3} = -V_0(0.005149)$$

And

$$19.23 V_{\pi 2} = -V_0(0.005149) \left(\frac{1}{2.6} + \frac{1}{50}\right) - \frac{V_0}{50} \quad (2)$$

$$= -V_0(0.02208)$$

$$\text{Or } V_{\pi 2} = -V_0(0.001148)$$

And

$$V_{\pi 1} = V_{in} - V_{\pi 2} = V_{in} + V_0(0.001148) \quad (4)$$

So

$$-V_0(0.001148) = [V_{in} + V_0(0.001148)] \left(\frac{101}{520}\right)(5.2) \quad (3)$$

$$-V_0(0.001148) - V_0(0.001159) = V_{in}(1.01)$$

$$\Rightarrow \underline{A_v = \frac{V_0}{V_{in}} = -438}$$

3.

From Eq. 10.105

$$A_v = \frac{-g_m^2}{\frac{1}{r_{o3}r_{o4}} + \frac{1}{r_{o1}r_{o2}}}$$

$$g_m = 2\sqrt{\left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)I_{D1}}$$

$$= 2\sqrt{(0.050)(20)(0.08)}$$

$$g_m = 0.5657 \text{ mA/V}$$

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{(0.02)(0.08)} = 625 \text{ K}$$

$$A_v = \frac{-(0.5657)^2}{\frac{1}{(625)^2} + \frac{1}{(625)^2}} = \frac{-0.3200}{2(0.00000256)}$$

$$A_v = -62,500$$

4.

a. For $v_1 = v_2 = 0$ and neglecting base currents

$$R_E = \frac{-0.7 - (-10)}{0.15} \Rightarrow \underline{R_E = 62 \text{ k}\Omega}$$

b.

$$A_d = \frac{v_{o2}}{v_d} = \frac{\beta R_C}{2(r_x + R_B)}$$

$$r_x = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.075} = 34.7 \text{ k}\Omega$$

$$A_d = \frac{(100)(50)}{2(34.7 + 0.5)} \Rightarrow \underline{A_d = 71.0}$$

$$A_{cm} = -\frac{\beta R_C}{r_x + R_B} \left[\frac{1}{1 + \frac{2R_E(1+\beta)}{r_x + R_B}} \right]$$

$$= -\frac{(100)(50)}{34.7 + 0.5} \left[\frac{1}{1 + \frac{2(62)(101)}{34.7 + 0.5}} \right] \Rightarrow \underline{A_{cm} = -0.398}$$

$$CMRR_{dB} = 20 \log_{10} \left| \frac{71.0}{0.398} \right| \Rightarrow \underline{CMRR_{dB} = 45.0 \text{ dB}}$$

c.

$$R_{id} = 2(r_x + R_B)$$

$$R_{id} = 2(34.7 + 0.5) \Rightarrow \underline{R_{id} = 70.4 \text{ k}\Omega}$$

Common-mode input resistance

$$R_{icm} = \frac{1}{2} [r_x + R_B + 2(1+\beta)R_E]$$

$$= \frac{1}{2} [34.7 + 0.5 + 2(101)(62)] \Rightarrow \underline{R_{icm} = 6.28 \text{ M}\Omega}$$