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Despread-ahead cyclic-prefix code division multiple access receiver with compressive sensing channel impulse response estimation

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Abstract: The conventional cyclic-prefix code division multiple access (CP-CDMA) system usually requires a chip-rate frequency-domain equaliser (FDE) at the cost of a tremendous discrete Fourier transform (DFT) size. To reduce the required DFT size in the CP-CDMA uplink receiver, this work studies a new symbol-rate equaliser moving the code despreader ahead of the FDE. The authors formulate the composite effect of the chip-rate channel impulse response (CIR) and code despreading as an equivalent symbol-rate channel model together with a small additive noise. Then, an iterative two-stage multiuser interference (MUI) cancellation algorithm can be employed with the decision feedback equalisation method. In addition, a novel compressive sensing (CS)-based interpolation method is proposed for compressible CIR estimation to be used in the MUI cancellation algorithm. With the CS method, the DFT size is dramatically reduced for channel estimation and no high-resolution interpolation kernels are required. The numerical results show that the new receiver is effective on MUI cancellation and only a couple of iterations are required to achieve a performance similar to the chip-rate equaliser.

1 Introduction

Although orthogonal frequency division multiplexing has been the most important modern wireless communication technology for the past two decades, the single-carrier wireless access technology is still attractive for uplink because of low peak to average power ratio [1]. In addition to the well-known single-carrier frequency division multiple access [2, 3] adopted in the 3GPP long-term evolution (LTE) standard [4], some advanced mutants to the direct sequence code division multiple access (DS-CDMA) scheme, for example, the multicarrier CDMA and cyclic-prefix CDMA (CP-CDMA), still remain high value because they are better suited to multipath channels for multiaccess broadband communications.

Owing to the inherent disadvantage of multiuser interference (MUI) in CDMA, a lot of research about MUI mitigation can be found in the literature [5]. For example, minimum mean square error (MMSE) [6] and zero-forcing (ZF) receiver [7] are typical methods to obtain a linear receiver. Parallel interference cancellation [8–10] and successive interference cancellation [11–13] are the well-known subtractive interference cancellation approach. CP-CDMA [14–17] is a promising alternative to original DS-CDMA because the frequency domain equaliser (FDE) [18–20] can be employed to combat multipath channels. However, a large fast Fourier transform (FFT) size due to the chip-rate-based computation still draws continuous study to reform CP-CDMA [21, 22]. Some recent works such as the block-spreading CDMA (BS-CDMA) performing code despreading ahead of the FDE, called despread-ahead structure here, can be found in the literatures [23–25]. Although the BS-CDMA scheme possibly provides MUI-free reception, a non-redundant precoding matrix for BS may need a considerable sub-block size, which is larger than the number of chip-rate CP length of the longest channel impulse response (CIR) among all users.

In this paper, a mathematical analysis is proposed to exploit the multiuser despread-ahead CP-CDMA receiver that does not employ the precoding matrix. From the analytic result, the composite effect of code despreading and the chip-rate CIR is formulated as a new equivalent symbol-rate CIR, which remains a linearly scaled property after the DFT such that a frequency-domain ZF or MMSE method can be employed to reduce the complexity of channel equalisation. For removing the MUI, a two-stage cancellation algorithm is adapted in our work. As in the numerical results, the iterative decision feedback FDE employed at the second stage can achieve a satisfying performance with only a couple of iterations.

For conventional CP-CDMA, the CIR can be directly estimated through the chip-rate equaliser. In the despread-ahead structure, only the symbol-rate CIR can be easily obtained at the despreaded output. As the subchannel is flat enough in frequency within the subcarrier bandwidth, the chip-rate CIR actually becomes compressible and can be properly approached by the downsampled version. Although a direct method for obtaining the chip-rate CIR is to interpolate the symbol-rate CIR, the accuracy is limited

by the interpolation kernel. Here, we develop a new compressive sensing (CS) [26, 27] based interpolation method for CIR estimation. In the literature, the CS method has been successfully used for sparse channel estimation [28-31] if the number of non-zero CIR coefficients is about as many as the size of the measurement vector [32]. Since the original CIR is not sparse in our case, the new CS-based channel estimation method is performed by applying the discrete cosine transform (DCT) to build a sparse basis from a modified LS formulation. In addition to the advantage of a smaller FFT size, this new algorithm also contributes to recovering the chip-rate CIR without using any high-resolution interpolation kernels. The proposed two-stage MUI cancellation (MUIC) scheme is simulated with reference to the mandatory IEEE 802.11 a/n/ ac specifications in the indoor environment for two and four users. The numerical results show that the new receiver has similar bit error rate (BER) performance to the conventional receiver, but with a lower complexity to implement the FDE.

The rest of this paper is organised as follows: Section 2 describes the system model in our framework. In Section 3, the mathematical model for the equivalent symbol-based FDE is proposed. The iterative two-stage MUIC algorithm and the MUI variance analysis are followed in this section. The new CS-based CIR estimation method is described in Section 4. Numerical results are explored in Section 5 and conclusion is drawn in Section 6.

2 CP-CDMA system model

Figs. 1a-c depict the transmitter and the receiver of a CP-CDMA system, respectively, where Fig. 1b shows the conventional receiver whereas Fig. 1c the despread-ahead

receiver. The main difference is that the conventional receiver first solves the channel response problem and then performs despreading after the FDE. In contrast, the new receiver first despreads the received signal before the FDE and then solves the channel response problem. Let N be the data block length and L_c the spreading code length. Since the FDE in the conventional receiver operates at chip rate, its FFT size is $L = NL_c$. The FFT size of the new receiver is N because of the symbol rate operation. Owing to the factor L_c for multiusers, the despread-ahead receiver owns the advantage of a reduced complexity in terms of the FFT size.

In our framework, we consider a *P*-user CP-CDMA system. Denote the symbol index by n, n = 0, 1, ..., N-1. Suppose the transmitted symbol $d_n^{(u)}$ is spreaded by the code vector $c^{(u)}$ where u is the user index and $1 \le u \le P$. If each symbol is spreaded into L_c chips, we can express $c^{(u)} = \left[c_0^{(u)}c_1^{(u)}\cdots c_{L_c-1}^{(u)}\right]$, where $c_i^{(u)} \in (1, -1)$ with $0 \le i \le L_c - 1$. Denote the input block vector by $d^{(u)} = \left[d_0^{(u)}d_1^{(u)}\cdots d_{N-1}^{(u)}\right]^T$. After spreading, we can write the spreaded block vector as

$$\boldsymbol{s}^{(u)} = \boldsymbol{d}^{(u)} \oplus \boldsymbol{c}^{(u)} = \left[s_0^{(u)} s_1^{(u)} \cdots s_{L-1}^{(u)} \right]^{\mathrm{T}}$$
(1)

where \oplus denotes the Kronecker product.

We assume an enough CP length N_g is added to the transmitted vector such that intersymbol interference can be avoided. We denote the CIR by $\boldsymbol{h}^{(u)} = \left[h_0^{(u)}h_1^{(u)}\cdots h_{L_g-1}^{(u)}\right]^T$, where L_g is the number of chip-rate multipath channel taps and $L_g < N_g$. Suppose the channel response is time-invariant within a data block and CP is perfectly removed at the



Fig. 1 System model

a Transmitter

b Conventional receiver

c New receiver

receiver, the received vector becomes

$$\boldsymbol{r} = \boldsymbol{h}^{(m)} \otimes \boldsymbol{s}^{(m)} + \sum_{\substack{u=1\\ u \neq m}}^{P} \boldsymbol{h}^{(u)} \otimes \boldsymbol{s}^{(u)} + \boldsymbol{z}$$
(2)

where \otimes denotes the circular convolution, $h^{(m)}$ is the CIR of the desired vector $s^{(m)}$, $s^{(u)}$ for $u \neq m$ is the undesired vector and z is the complex additive white Gaussian noise (AWGN) vector. The despreaded signal for user m can be expressed as

$$y_{n}^{(m)} = \frac{1}{L_{c}} \sum_{t=nL_{c}}^{(n+1)L_{c}-1} r_{t} c_{t}^{(m)}$$

$$= \underbrace{\frac{1}{L_{c}} \sum_{t=nL_{c}}^{(n+1)L_{c}-1} (h_{t}^{(m)} \otimes s_{t}^{(m)}) c_{t}^{(m)}}_{(1)}}_{(1)}$$

$$+ \underbrace{\frac{1}{L_{c}} \sum_{t=nL_{c}}^{P} \sum_{(1)}^{(n+1)L_{c}} (h_{t}^{(u)} \otimes s_{t}^{(u)}) c_{t}^{(m)} + z_{n}}_{(3)}$$
(3)

where the first term is the desired received signal, the second term is MUI and the third term is a complex AWGN variable with

$$z_n = \frac{1}{L_c} \sum_{t=nL_c}^{(n+1)L_c - 1} z_t c_t^{(m)}$$
(4)

Proposed CP-CDMA receiver with MUIC 3

3.1 Analysis of the desired received signal

Let us begin with the analysis of the desired received signal. Denoting the circular convolution of $h_t^{(m)}$ and $s_t^{(m)}$ by

$$h_t^{(m)} \otimes s_t^{(m)} = \sum_{j=0}^{L-1} h_{[i-j]_L}^{(m)} s_j^{(m)}$$
(5)

where $[\cdot]_L$ indicates $[\cdot]$ modulo L operation, we can rewrite the first term of (3) as

$$y_{n}^{'(m)} = \frac{1}{L_{c}} \sum_{t=nL_{c}}^{(n+1)L_{c}-1} \left(\sum_{j=0}^{L-1} h_{[t-j]_{L}}^{(m)} s_{j}^{(m)} \right) c_{t}^{(m)}$$

$$= \frac{1}{L_{c}} \sum_{t=0}^{L_{c}-1} \sum_{j=0}^{L-1} h_{[nL_{c}+t-j]_{L}}^{(m)} s_{j}^{(m)} c_{t}^{(m)}$$
(6)

Changing the variable $j = lL_c + k$ for l = 0, 1, ..., N-1 and $z = 0, 1, ..., L_c - 1$, we have

$$y_{n}'(m) = \frac{1}{L_{c}} \sum_{t=0}^{L_{c}-1} \sum_{l=0}^{N-1} \sum_{k=0}^{L_{c}-1} h_{[nL_{c}+t-lL_{c}-k]_{L}}^{(m)} s_{lL_{c}+k}^{(m)} c_{t}^{(m)}$$

$$= \frac{1}{L_{c}} \sum_{t=0}^{L_{c}-1} \sum_{l=0}^{N-1} \sum_{k=0}^{L_{c}-1} h_{[(n-l)L_{c}+(t-k)]_{L}}^{(m)} s_{lL_{c}+k}^{(m)} c_{t}^{(m)}$$
(7)

Using the relationship $s_{lL_c+k}^{(m)} = d_l^{(m)} c_k^{(m)}$ in (1), (7) becomes

$$y_{n}^{\prime(m)} = \sum_{l=0}^{N-1} \underbrace{\frac{1}{L_{c}} \sum_{t=0}^{L_{c}-1} \sum_{k=0}^{L_{c}-1} c_{l}^{(m)} c_{k}^{(m)} h_{[(n-l)L_{c}+(t-k)]_{L}}^{(m)}}_{\mathcal{M}_{n-l}^{(m)}} d_{l}^{(m)}}$$
(8)

Dividing the term $\mathcal{M}_{n-l}^{(m)}$ as denoted in (8) into two parts for the summation over *t*, we have:

(i) When t = k

$$\mathcal{M}_{n-l,t=k}^{(m)} = \frac{1}{L_{c}} \sum_{k=0}^{L_{c}-1} (c_{k}^{(m)})^{2} h_{[(n-l)L_{c}]_{L}}^{(m)}$$

$$= \tilde{h}_{[n-l]_{N}}^{(m)}$$
(9)

where $(c_k^{(m)})^2 = 1$ and $\tilde{h}_{[n-l]_N}^{(m)}$ can be read as the equivalent symbol-rate CIR. (ii) When $t \neq k$

$$\mathcal{M}_{n-l,t\neq k}^{(m)} = \frac{1}{L_{c}} \sum_{\substack{t=0\\t\neq k}}^{L_{c}-1} \sum_{k=0}^{L_{c}-1} c_{t}^{(m)} c_{k}^{(m)} h_{[(n-l)L_{c}+(t-k)]_{L}}^{(m)}$$
(10)

Considering the case $c_t^{(m)}$ is nearly a white sequence and L_c is large enough, (10) is very close to zero. Using the identity $\mathcal{M}_{n-l}^{(m)} = \mathcal{M}_{n-l,l=k}^{(m)} + \mathcal{M}_{n-l,l\neq k}^{(m)}$ and (8)–(10), we obtain

$$y_{n}^{\prime(m)} = \sum_{l=0}^{N-1} \tilde{h}_{[n-l]_{N}}^{(m)} d_{l}^{(m)} + \sum_{l=0}^{N-1} M_{n-l,t\neq k}^{(m)} d_{l}^{(m)}$$

$$= \tilde{h}_{n}^{(m)} \otimes d_{n}^{(m)} + \bar{w}_{n}^{(m)}(h, c, d)$$
(11)

where $\bar{w}_{n}^{(m)}(h, c, d) = \sum_{l=0}^{N-1} \mathcal{M}_{n-l, t \neq k}^{(m)} d_{l}^{(m)}$. The DFT output of $y_{n}^{(m)}$ is

$$Y_k^{(m)} = \mathrm{DFT}_N \{Y_n^{(m)}\}$$

= $\widetilde{H}_k^{(m)} D_k^{(m)} + \overline{W}_k^{(m)}(h, c, d)$ (12)

where $k = 0, 1, ..., N-1, \widetilde{H}_k^{(m)} = \text{DFT}_N \{ \widetilde{h}_n^{(m)} \}, D_k^{(m)} =$ $\mathrm{DFT}_{N}\left\{d_{n}^{(m)}\right\}$ and $\overline{W}_{k}^{(m)}(h, c, d) = \mathrm{DFT}_{N}\left\{\bar{w}_{n}^{(m)}(h, c, d)\right\}$, accounting for a non-zero value of $\mathcal{M}_{n-l,t\neq k}^{(m)}$. Here, we can see that the value of $\overline{W}_k^{(m)}(h, c, d)$ is complicated and affected by the CIR *h*, spreading code *c* and transmitted symbol d.

To see the effect of $\overline{W}_k^{(m)}(h, c, d)$, Fig. 2 shows the least squares (LS) channel estimation result for 64-point DFT, with a typical multipath channel without taking the AWGN into consideration. Here, c1 is a randomly chosen Gold sequence of length 15 and c2 a Gold sequence of length 31. The preambles p1 and p2 are randomly generated binary phase shift keying signals. It is not surprising that the estimated channel values for different patterns have slight variation even when the noise is set to zero because $\overline{W}_{k}^{(m)}(h, c, d)$ is not actually zero. However, the estimated results are rather close to the real channel.

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Fig. 2 Initial channel estimation for different pilot and spreading code patterns

3.2 Two-stage MUIC

In (12), $D_k^{(m)}$ can be obtained from $Y_k^{\prime(m)}$ by using linear filtering methods such as LS error and MMSE once the channel response is given. However, the performance will be improved when the MUI signal in the second term of (3) is properly cancelled. The MUIC algorithm is employed in front of the code despreader in order to reduce residual MUI through the despreading operation.

(i) Stage 1: In this stage, the effect of MUI and noises are roughly reduced by the despreader. A frequency-domain MMSE equaliser then yields an initial estimate of $d^{(u)}$, denoted as $\hat{d}^{(u),0}$. From (3) and (12), the DFT output of $y_n^{(u)}$ is

$$Y_{k}^{(u),0} = \widetilde{H}_{k}^{(u)} D_{k}^{(u)} + \widetilde{W}_{k}^{(u)}$$
(13)

where $\widetilde{W}_{k}^{(u)} = Z_{k,\text{MUI}}^{(u)} + \overline{Z}_{k}^{(u)}$, $Z_{k,\text{MUI}}^{(u)}$ represents the DFT output of the MUI and $\overline{Z}_{k}^{(u)} = \overline{W}_{k}^{(u)}(h, c, d) + Z_{k}$ with $Z_{k} = \text{DFT}_{N}Z_{n}$. The MMSE equaliser for this stage can be found by giving the coefficients

$$G_{1,k}^{(u)} = \frac{\widetilde{H}_{k}^{(u)*}}{\left|\widetilde{H}_{k}^{(u)}\right|^{2} + \left(\sigma_{\widetilde{W}}^{2}(u)/\sigma_{d}^{2}\right)}$$
(14)

where (·)* indicates complex conjugate, $\sigma_{\widetilde{W}}^2(u) = \sigma_{\text{MUI}}^2(u) + \sigma_{\widetilde{Z}}^2(u)$ is the noise power for user u, $\sigma_{\widetilde{W}}^2(u) = E\left[|\widetilde{W}_k^{(u)}|^2\right]$, $\sigma_{\text{MUI}}^2(u) = E\left[|\overline{Z}_{k,\text{MUI}}^{(u)}|^2\right]$ and σ_d^2 represents the signal power. Here, the MUI is simply viewed as an additional noise such that a longer length spreading code has better capability to suppress the MUI. (ii) *Stage 2*: The initial estimate $\hat{d}^{(u),0}$ at Stage 1 or the decision feedback $\hat{d}^{(u)}$ in an iterative loop is used to yield $\hat{s}^{(u)}$. Once the channel estimate $\hat{h}^{(u)}$ is given, we use (2) to calculate the signal vector $\mathbf{v}^{(m)}$

$$\boldsymbol{v}^{(m)} = \boldsymbol{r} - \sum_{\substack{u=1\\u\neq m}}^{P} \hat{\boldsymbol{h}}^{(u)} \otimes \hat{\boldsymbol{s}}^{(u)}$$

$$= \boldsymbol{h}^{(m)} \otimes \boldsymbol{s}^{(m)} + \boldsymbol{g}^{(m)} + \boldsymbol{z}$$
(15)

where $\mathbf{g}^{(m)}$ denotes the residual MUI for user m and $\mathbf{g}^{(m)} = \sum_{\substack{u=1\\u\neq m}}^{P} \mathbf{h}^{(u)} \otimes \mathbf{s}^{(u)} - \sum_{\substack{u=1\\u\neq m}}^{P} \hat{\mathbf{h}}^{(u)} \otimes \hat{\mathbf{s}}^{(u)}$ After removing MUI and code despreading, from (15) we can express the despreader's output as

$$y_n^{(m)} = \frac{1}{L_c} \sum_{t=nL_c}^{(n+1)L_c - 1} v_t^{(m)} c_t^{(m)}$$

$$= \tilde{h}_n^{(m)} \otimes d_n^{(m)} + w_n^{(m)}$$
(16)

where $w_n^{(m)}$ is the noisy term with $w_n^{(m)} = (1/L_c)$ $\sum_{t=nL_c}^{(n+1)L_c-1} g_t^{(m)} c_t^{(m)} + \bar{w}_n^{(m)}(h, c, d) + z_n$ At the DFT output, we have

$$Y_k^{(m)} = \tilde{H}_k^{(m)} D_k^{(m)} + W_k^{(m)}$$
(17)

where $W_k^{(m)} = \text{DFT}_N\{v_n^{(m)}\}$. As a sequence, the new noise is reduced by introducing the residual MUI effect $g^{(m)}$. Hence, the noise variance of $W_k^{(m)}$ is far less than that of $\widetilde{W}_k^{(m)}$. Finally, we have the MMSE equaliser for Stage 2 with coefficients

$$G_{2,k}^{(m)} = \frac{\widetilde{H}_{k}^{(m)*}}{\left|\widetilde{H}_{k}^{(m)}\right|^{2} + \left(\sigma_{W}^{2}(m)/\sigma_{d}^{2}\right)}$$
(18)

where $\sigma_W^2(m) = E\left[|W_k^{(m)}|^2\right]$.

Iterative MUIC algorithm: In Stage 2, only the residual MUI effect is involved rather than the MUI effect. As a result, the estimate $\hat{d}^{(u)}$ obtained from Stage 2 is more accurate than that obtained from Stage 1. Since the calculation of MUI depends on real transmitted data, the iterative feedback from $\hat{d}^{(u)}$ can be applied to reduce residual MUI. We use the following iterative MUIC algorithm

Initialisation: Set
$$j = 0$$
 and $\hat{s}^{(u),j} = \left\{ \hat{d}_n^{(u),j} c^{(u)} \right\}_{n=0}^{N-1}$
for $u = 1, 2, ..., P$
Loop $j = j + 1$
Set $v^{(m),j} = r - \sum_{\substack{u=1\\u \neq m}}^{P} \hat{h}^{(u)} \otimes \hat{s}^{(u),j-1}$
 $\hat{s}^{(u),j} = \left\{ \hat{d}_n^{(u),j} c^{(u)} \right\}_{n=0}^{N-1}$

Go back to Loop

Let index *j* be the iteration number and set the initial value j = 0. The termination of the iterative algorithm can be determined by a tradeoff between system performance and computational loading. In our numerical cases, a nearly satisfying performance can be achieved after a couple of iterations.

3.3 MUI variance analysis

The MUIC algorithm requires the information of some noise variances. To apply the MMSE equaliser in Stage 1, we need to explore the variance in (14) for the *m*th user. Let $\widetilde{W}^{(m)} = \left[\widetilde{W}_{0}^{(m)}\widetilde{W}_{1}^{(m)}\cdots\widetilde{W}_{N-1}^{(m)}\right]^{\mathrm{T}}$, $Z_{\mathrm{MUI}}^{(m)} = \left[Z_{0,\mathrm{MUI}}^{(m)}Z_{1,\mathrm{MUI}}^{(m)}\cdots Z_{N-1,\mathrm{MUI}}^{(m)}\right]^{\mathrm{T}}$, $\overline{W}^{(m)}(h, c, d) = \left[\overline{W}_{0}^{(m)}(h, c, d)\overline{W}_{1}^{(m)}(h, c, d)\cdots \overline{W}_{N-1}^{(m)}(h, c, d)\right]^{\mathrm{T}}$ and $Z = [Z_{0}Z_{1}\cdots Z_{N-1}]^{\mathrm{T}}$. Assuming $Z_{\mathrm{MUI}}^{(m)}$, $\overline{W}^{(m)}(h, c, d)$ and Z are zero-mean and mutually uncorrelated, the variance of $\widetilde{W}^{(m)}$ is

$$\sigma_{\widetilde{W}}^2(m) = \sigma_{\text{MUI}}^2(m) + \sigma_{\widetilde{W}}^2(m) + \sigma_Z^2$$
(19)

where $\sigma_{\text{MUI}}^2(m) = E \Big[\| \mathbf{Z}_{\text{MUI}}^{(m)} \|^2 \Big] / N$, $\sigma_{\overline{W}}^2(m) = E \Big[\| \overline{\mathbf{W}}^{(m)}(m) + C(m) \|^2 \Big] / N$ and $\sigma_Z^2 = E [\| \mathbf{Z} \|^2] / N$.

From (3), $\sigma_{MUI}^2(m)$ can be further evaluated as follows. Let

$$z_{n,\text{MUI}}^{(m)} = \frac{1}{L_c} \sum_{\substack{u=1\\u\neq m}}^{P} \sum_{t=nL_c}^{(n+1)L_c} (h_t^{(u)} \otimes s_t^{(u)}) c_t^{(m)}$$
(20)

and $\mathbf{z}_{MUI}^{(m)} = \left[z_{0,MUI}^{(m)} z_{1,MUI}^{(m)} \cdots z_{N-1,MUI}^{(m)} \right]^{T}$. Denote by \mathbf{F}_{N} the $N \times N$ normalised DFT matrix, then we have $\mathbf{F}_{N}^{H} \mathbf{F}_{N} = \mathbf{I}_{N}$, where \mathbf{I}_{N} is the $N \times N$ identity matrix. We can express the DFT output of the second term in (3) as $\mathbf{Z}_{MUI}^{(m)} = \mathbf{F}_{N} \mathbf{z}_{MUI}^{(m)}$, the variance being (see (21))

Using the property

$$\sum_{\substack{u=1\\t\neq m}}^{P} \sum_{\substack{v=1\\v\neq m}}^{P} \sum_{t=nL_{c}}^{(n+1)L_{c}} \sum_{k=nL_{c}}^{(n+1)L_{c}} E\left[(\cdot)c_{t}^{(u)}c_{k}^{(v)}\right]$$

$$\simeq \begin{cases} \sum_{\substack{u=1\\u\neq m}}^{P} \sum_{t=nL_{c}}^{(n+1)L_{c}} E\left[\cdot\right], & \text{when } u = v \text{ and } t = k \end{cases}$$
(22)

we have the following approximation

$$\sigma_{\text{MUI}}^{2}(m) = \frac{1}{NL_{c}^{2}} \sum_{n=0}^{N-1} \sum_{u=1}^{P} \sum_{t=nL_{c}}^{(n+1)L_{c}} E\Big[\left|h_{t}^{(u)} \otimes s_{t}^{(u)}\right|^{2}\Big]$$

$$= \frac{\sigma_{d}^{2}}{L_{c}^{2}} \sum_{u=1u\neq m}^{P} \sum_{k=0}^{N-1} E\Big[\left|H^{(u)}(k)\right|^{2}\Big]$$
(23)

where $H_k^{(u)} = \text{DFT}_N \{h_n^{(u)}\}$ and the discrete-time Parseval's

energy theorem is applied with that $\sum_{n} E[|x_{n}|^{2}] = \sum_{k} E[|X_{k}|^{2}]$, where $X_{k} = \text{DFT}_{N}x_{n}$. Note that $\sigma_{\widetilde{W}}^{2}(m)$ is dominated by $\sigma_{\text{MUI}}^{2}(m)$.

The next variance of interest to be used in the proposed MUIC method is $\sigma_W^2(m)$ in (18). Let $w_n^{\prime(m)} = (1/L_c) \sum_{t=nL_c}^{(n+1)L_c-1} g_t^{(m)} c_t^{(m)}$ and $w'(m) = \left[w_0^{\prime(m)} w_1^{\prime(m)} \cdots w_{N-1}^{\prime(m)}\right]^T$. The DFT output of $w'^{(m)}$ is $W'^{(m)} = F_N w'^{(m)}$. Define $W^{(m)} = \left[W_0^{(m)} W_1^{(m)} \cdots W_{N-1}^{(m)}\right]^T$, the variance of $W^{(m)}$ can be expressed as

$$\sigma_W^2(m) = \sigma_{W'}^2(m) + \sigma_{\overline{W}}^2(m) + \sigma_Z^2$$
(24)

where

$$\sigma_{W'}^{2}(m) = \frac{1}{N} E \left[\left(\boldsymbol{F}_{N} \boldsymbol{w}^{\prime(m)} \right)^{\mathrm{H}} \left(\boldsymbol{F}_{N} \boldsymbol{w}^{\prime(m)} \right) \right]'$$
$$= \frac{1}{NL_{\mathrm{c}}^{2}} \sum_{n=0}^{N-1} \sum_{t=nL_{\mathrm{c}}}^{(n+1)L_{\mathrm{c}}} \sum_{k=nL_{\mathrm{c}}}^{(n+1)L_{\mathrm{c}}} E \left[\left(g_{t}^{(m)} \right)^{*} g_{k}^{(m)} \right] c_{t}^{(m)} c_{k}^{(m)}$$
(25)

Using the spreading code property in (22) and assuming ideal channel estimation, $\hat{\boldsymbol{h}}^{(u)} = \boldsymbol{h}^{(u)}$, from (15) and (25) we have

$$\sigma_{W'}^{2}(m) = \frac{1}{NL_{c}^{2}} \sum_{n=0}^{N-1} \sum_{\substack{u=1\\u\neq m}}^{P} \sum_{t=nL_{c}}^{(n+1)L_{c}} E\Big[\left|h_{t}^{(u)}\otimes\left(s_{t}^{(u)}-\hat{s}_{t}^{(u)}\right)\right|^{2}\Big]$$
$$= \frac{\sigma_{\Delta}^{2}}{L_{c}^{2}} \sum_{\substack{u=1\\u\neq m}}^{P} \sum_{k=0}^{N-1} E\Big[\left|H^{(u)}(k)\right|^{2}\Big]$$
(26)

where σ_{Δ}^2 is the variance of $\Delta = d_n^{(u)} - \hat{d}_n^{(u)}$. It is obvious that if the decision output $\hat{s}_t^{(u)}$ is almost a replica of $s_t^{(u)}$, $\sigma_{W'}^2(m)$ approaches zero and the variance $\sigma_W^2(m)$ is dominated by σ_Z^2 . Besides, comparing (23) and (26), in general, we have $\sigma_{MUI}^2(m) > \sigma_{W'}^2(m)$, that is, $\sigma_{\tilde{W}}^2(m) > \sigma_W^2(m)$, indicating that the receiver performance with the second stage is much improved.

4 CS-based modified LS method for CIR estimation

Considering the CIR is compressible and can be reconstructed from its downsampled version, one can capture the useful channel information content embedded in a condensed sparse signal and a compressive sensing (or measurement)

$$\sigma_{\text{MUI}}^{2}(m) = \frac{1}{N} E \left[\left(\boldsymbol{F}_{N} \boldsymbol{z}_{\text{MUI}}^{(m)} \right)^{\text{H}} \left(\boldsymbol{F}_{N} \boldsymbol{z}_{\text{MUI}}^{(m)} \right) \right]$$

$$= \frac{1}{NL_{c}^{2}} \sum_{n=0}^{N-1} \sum_{u=1}^{P} \sum_{v=1 \atop u \neq m}^{P} E \left[\left(\sum_{t=nL_{c}}^{(n+1)L_{c}} \left(h_{t}^{(u)} \otimes \boldsymbol{s}_{t}^{(u)} \right) \boldsymbol{c}_{t}^{(m)} \right)^{*} \left(\sum_{k=nL_{c}}^{(n+1)L_{c}} \left(h_{k}^{(v)} \otimes \boldsymbol{s}_{k}^{(v)} \right) \boldsymbol{c}_{k}^{(m)} \right) \right]$$

$$= \frac{1}{NL_{c}^{2}} \sum_{n=0}^{N-1} \sum_{u=1 \atop u \neq m}^{P} \sum_{v=1 \atop v \neq m}^{P} \sum_{t=nL_{c}}^{(n+1)L_{c}} E \left[\left(h_{t}^{(u)} \otimes \boldsymbol{s}_{t}^{(u)} \right)^{*} \left(h_{k}^{(v)} \otimes \boldsymbol{s}_{k}^{(v)} \right) \right]$$
(21)

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system can be used to recover the CIR with far fewer samples than the chip-rate FFT size. The CS theory gives convex optimisation to construct the full-length signal from a small and efficient amount of collected measurements.

4.1 Brief CS theory review

Suppose x is a real-valued and one-dimensional vector in \mathbb{R}^M , which is to be reconstructed from a measurement vector f in \mathbb{R}^Q from the sensing system $f = \Phi x$, where Φ is a $Q \times M$ sensing matrix and Q < M. The CS theory states that if x has a sparse representation in some transform basis, say Ψ , that is, $x = \Psi u$, where Ψ is an $M \times T$ sparse transform matrix and the $T \times 1$ transformed vector u has K non-zero coefficients and T - K zeros or, at least, very small values compared to the K non-zero coefficients. The signal x is compressible as u is K-sparse.

One critical problem in the CS method is to design a stable sensing matrix Φ such that the signal x can be recovered without the damage of the underdetermined representation. In a realistic application, the noisy component usually presents in the sensing system. Denoting the sensing noise by *e*, the noisy measurement equation becomes

$$f = \Theta u + e \tag{27}$$

where $\Theta = \Phi \Psi$. The well-conditioned requirement of Θ for general robustness of CS is that Θ satisfies the restricted isometry property (RIP) [32], that is, $1 - \delta \le || \Theta u ||_2 / ||$ $u ||_2 \le 1 + \delta$ for some $\delta > 0$. It can be shown [26] that a $Q \times M$ random Gaussian matrix has the RIP with high probability if $Q \ge cK \log(M/K)$ with *c* a small constant. Then, the signal **x** can be recovered by solving **u** with minimum ℓ_1 norm

$$\hat{\boldsymbol{u}} = \arg \min \|\boldsymbol{u}\|_1 \quad \text{subject to} \\ \|\boldsymbol{\Theta}\boldsymbol{u} - \boldsymbol{f}\|_2 \le \epsilon$$
(28)

where ϵ is a specified parameter to bound the amount of noise, and then, $\hat{x} = \Psi \hat{u}$. Since the RIP theory is beyond the scope of this work, we simply omit the issue here. More discussion can be referred to [26].

4.2 Proposed CIR estimation method

In the CP-CDMA system, the CIR information is required for MUIC at the receiver. Compared to the MMSE CIR estimation method, 1.S is more simple for realisation. Before proposing a new and lower-complexity CIR estimation algorithm, we first adapt the LS method to this problem. Notice that $h^{(u)}$ is the $L_g \times 1$ chip-rate complex CIR vector to be estimated for user u. Denote by $F_{p \times q}$ the $p \times q$ DFT matrix with coefficients defined by $[F]_{k, n} = p^{-1/2} \exp(-i2\pi kn/p)$, where $p \ge q$ for k=0, 1, ..., p-1 and n=0, 1, ..., q-1. Once the chip-rate DFT output vector $V_{M_c \times 1}^{(u)}$, where the subscript $NL_c \times 1$ represents the number of elements, is given, the conventional CP-CDMA estimates $h^{(u)}$ by solving

$$\boldsymbol{Y}_{NL_{c}\times1}^{(u)} = \boldsymbol{D}\boldsymbol{F}_{NL_{c}\times L_{g}}\boldsymbol{h}^{(u)} + \boldsymbol{W}_{NL_{c}\times1}^{(u)}$$
(29)

where D is an $NL_c \times NL_c$ diagonal matrix with pilots as its diagonal entries and $W_{NL_c \times 1}^{(u)}$ represents the $NL_c \times 1$ noise

vector. Without loss of generality, we can set D the identity matrix.

In our framework model, the despreading in front of the DFT reduces the required number of input samples for DFT with a downsampling rate from the CIR. Suppose the new DFT output vector in the despread-ahead CP-CDMA receiver is reduced to $Y_{Q\times 1}^{(u)}$, that is, at a downsampling rate λ where $\lambda = NL_c/Q$. (To choose the downsampling rate as an integer, we usually set the value of Q such that $\lambda = NL_c/Q$ is an integer. For instance, when N = 64 and $L_c = 15$. Q = 64 is used for the downsampling rate 15, Q = 192 for the downsampling rate 5 and Q = 320 for the downsampling rate 3.) For simplicity, let us consider the despreader as an ideal downsampler. Then, (29) can be reduced as

$$\boldsymbol{Y}_{Q\times 1}^{(u)} = \boldsymbol{F}_{Q\times \lfloor L_g/\lambda \rfloor} \mathcal{D}\boldsymbol{h}^{(u)} + \boldsymbol{W}_{Q\times 1}^{(u)}$$
(30)

where \mathcal{D} is an $[L_g/\lambda] \times L_g$ downsampling matrix with entries

$$\left[\mathcal{D}\right]_{i,j} = \begin{cases} 1, & j = \lambda i + 1\\ 0, & \text{otherwise} \end{cases}$$
(31)

for $i=0, 1, ..., \lfloor L_g/\lambda \rfloor - 1$. In consequence, the CIR at the downsampling rate λ can be found by the following LS solution to $h^{(u)}$ with subscript *j* defined in (31)

$$[\hat{\boldsymbol{h}}_{\text{LS}}^{(u)}]_{j} = \left[\left(\boldsymbol{F}_{\mathcal{Q} \times \left\lfloor L_{g} / \lambda \right\rfloor} \mathcal{D} \right)^{\perp} \boldsymbol{Y}_{\mathcal{Q} \times 1}^{(u)} \right]_{j}$$
(32)

where $(\cdot)^{\perp}$ represents taking psuedo inverse. Once the downsampled CIR is obtained, the chip-rate CIR can be approximated through some interpolation methods such as linear interpolation, second-order polynomial interpolation, spline interpolation, and so on.

Although the conventional LS formulation (30) is established, it cannot be used for CS because not all CIR coefficients are linked to the DFT output. Some random values on the CIR coefficients out of the downsampled points will be resulted if (30) is employed. Towards a more accurate insight into despreading, a smoothing model over the downsampled points is proposed to replace the ideal downsampler \mathcal{D} in (30). We define by S the $\lfloor L_g/\lambda \rfloor \times L_g$ smoothing matrix with entries

$$[\mathcal{S}]_{i,j} = \begin{cases} 1/\lambda, & j = \lambda i + 1, \ \lambda i + 2, \ \dots, \ \lambda(i+1) \\ 0, & \text{otherwise} \end{cases}$$
(33)

Substituting \mathcal{D} in (30) with \mathcal{S} , we have

$$\boldsymbol{Y}_{Q\times 1}^{(u)} = \boldsymbol{F}_{Q\times \left\lfloor L_g/\lambda \right\rfloor} \mathcal{S}\boldsymbol{h}^{(u)} + \boldsymbol{W}_{Q\times 1}^{(u)}$$
(34)

and the modified LS solution to the chip-rate $h^{(u)}$ can be fully approached by

$$\hat{\boldsymbol{h}}_{\text{MLS}}^{(u)} = \left(\boldsymbol{F}_{\mathcal{Q} \times \left\lfloor L_g / \lambda \right\rfloor} \mathcal{S}\right)^{\perp} \boldsymbol{Y}_{\mathcal{Q} \times 1}^{(u)}$$
(35)

Although the L_g -tap CIR can be solved with (35), the smoothing model S only provides the uniform weighting function for each group of λ points from $\boldsymbol{h}^{(u)}$ such that $\hat{\boldsymbol{h}}_{\text{MLS}}^{(u)}$ can be viewed as the nearest neighbour interpolation

of $\left[\hat{\boldsymbol{h}}_{LS}^{(u)}\right]_{j}$. A more appealing interpolation method based on CS to solve (34) is thus considered in our work.

Revisit the real-valued CS formulation described in Section 4.1. Since $\mathbf{h}^{(u)}$ is in \mathbb{C}^{L_g} , we let $x = [\operatorname{Re}\{\mathbf{h}^{(u)}\}^{\mathrm{T}}\operatorname{Im}\{\mathbf{h}^{(u)}\}^{\mathrm{T}}]^{\mathrm{T}}$, where $\operatorname{Re}\{\cdot\}$ and $\operatorname{Im}\{\cdot\}$ represent taking real part and imaginary part, respectively. Then, $M = 2L_g$ and x becomes an $M \times 1$ real-valued vector. Assume the Q/2-point FFT output provides the noisy measurement vector $\mathbf{Y}^{(u)}$, then f = $[\operatorname{Re}\{\mathbf{Y}^{(u)}\}^{\mathrm{T}}\operatorname{Im}\{\mathbf{Y}^{(u)}\}^{\mathrm{T}}]^{\mathrm{T}}$, where f is a $Q \times 1$ real-valued vector. Define $\Phi_c = \mathbf{F}_{Q/2 \times \lfloor L_g/\lambda \rfloor} S$ from (34), the sensing matrix which links \mathbf{x} to f becomes

$$\Phi = \begin{bmatrix} \operatorname{Re}\{\Phi_{c}\} & -\operatorname{Im}\{\Phi_{c}\}\\ \operatorname{Im}\{\Phi_{c}\} & \operatorname{Re}\{\Phi_{c}\} \end{bmatrix}$$
(36)

It is worth noting that the complex-valued channel gains are not affected by the decomposition of real-valued formulation. The real-valued formulation is to satisfy the CS requirements defined in [33] and is actually equivalent to the complex-valued one.

The CIR property affects the determination of the transform basis used for sparse representation of x; here, assuming the power profile of the CIR is exponentially decayed and the chip-rate samples satisfy that the chip-rate CIR can be approximated by means of interpolation from its downsampled versions. For example, this multipath channel condition is usually adopted in typical indoor wireless local area network (WLAN) environments. Based on the above assumption, the cosine basis used in DCT is considered in our work since cosine functions are much more efficient than sine functions for signal compression from compressible CIR. Denote by $T_{p \times q}$ the $p \times q$ DCT matrix with entries $[T]_{i,j} = \sqrt{2/p} \cdot \cos((2j+1)i\pi/2p)$ where $p \ge q$, i=0, 1, ..., p-1, and j=0, 1, ..., q-1. Since x consists of real part and imaginary part of the chip-rate complex CIR, we represent $u = \mathcal{T}x$ where \mathcal{T} is a $T \times M$ matrix and

$$\mathcal{T} = \begin{bmatrix} \mathbf{T}_{\frac{T}{2} \times \frac{M}{2}} & \mathbf{0} \\ \mathbf{0} & \mathbf{T}_{\frac{T}{2} \times \frac{M}{2}} \end{bmatrix}$$
(37)

where 0 is a $T/2 \times M/2$ zero matrix. Then, the sparse transform matrix is $\Psi = \mathcal{T}^{\perp}$.

5 Simulation results

5.1 CS-based CIR estimation

Consider the chip-rate CIR can be properly approached by the interpolation of the downsampled CIR. Then, we exploit the chip-rate CIR from the compressive property of the downsampled CIR or its DFT measurements. Here, we study two different kinds of multipath channel models that are widely used in WLAN indoor environments and 3GPP wireless communications. The typical indoor multipath channel is usually described by an exponentially decaying power profile, for example, if the first 8 significant taps are considered and normalised for 64-point DFT, we can set the discrete-time CIR $h(n) \sim CN(0, \sigma^2(n))$, where $\sigma^2(n) = e^{-n/2}$ for n = 0, 1, ..., 7. The average root mean square (rms) delay spread is 1.6436 symbols, that is, 82.18 ns with 50 ns symbol duration as specified in IEEE 802.11 a/n/ac standards. For 3GPP wireless communications, the typical urban and non-hilly (TU) channel model suggested in COST 207 [34] also uses the exponential profile with one decaying peak as $e^{-\tau/1\mu s}$ for $0 \le \tau < 7 \mu s$. The LTE standard realises the FDE with 15 KHz subcarrier spacing for 256-point DFT, that is, the symbol duration is 260 ns with 2.5 MHz transmission bandwidth. Then, the average rms delay spread is 3.2537 symbols provided that the first 16 significant taps are considered and normalised, that is, 0.846 µs in this case.

To evaluate the proposed CIR estimation method for the two channel models, we compare the mean square error (MSE) of estimation for different methods with a variety of Q as the number of sensing samples per block form the measurement vector. To simplify the simulation, a complex cubic spline interpolation function is used to construct the original chip-rate CIR and is then also used to reconstruct the CIR from the LS solution as the interpolation performance bound. Here, we assume that the $Q \times 1$ DFT output vector is contaminated by an additive complex white Gaussian noise vector with 10 dB channel power to noise ratio.

Fig. 3a shows the results with $L_c = 15$ for the indoor channel model. The pseudo-inverse (PI) interpolation method is referred to (35), which is obviously worse than the linear interpolation and the CS interpolation methods based on the downsampled CIR under 100 runs Monte-Carlo simulation. Here, the PI interpolation result is used as the initial value for CS recovery that is implemented with the $11qc_{10}$ program in the well-known ℓ_1 -magic toolbox [33], in which the iteration stop parameter is suggested as $\epsilon = \sigma \sqrt{Q}$, where σ is the noise standard deviation for a normalised channel gain. Other CS reconstruction algorithms such as matching pursuit, orthogonal matching pursuit, and so on may provide better performance, however, the discussion about CS algorithms is not the purpose of this work and can be referred to [35]. In our simulation case, Q = 128 can lead to an almost saturated MSE performance close to the cubic spline bound. From these results, we can see that the CS method is effective to estimate the chip-rate CIR through $2 \times$ oversampled FFT measurements; in contrast, $15 \times$



Fig. 3 *CIR estimation MSE against different* Q *with* $L_c = 15$ *and 10 dB channel to noise power ratio a* Indoor WLAN channel model *b* COST 207 TU channel model

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Fig. 4 *Two reconstructed chip-rate CIR examples with different interpolation methods*

a Channel example generated by the WLAN indoor channel model *b* Channel example generated by the 3GPP COST 207 TU channel model

oversampling is equivalent to the chip-rate FFT processing with $L_c = 15$. Fig. 3b shows the results with $L_c = 15$ for the 3GPP TU channel model, where Q = 256 leads to an almost saturated MSE performance and this indicates that the same 256-point FFT size can be used for CIR estimation with the proposed CIR estimation method in this case. Figs. 4a and b show the interpolated results of two CIR examples generated by the WLAN indoor channel and the 3GPP COST 207 TU channel models, respectively, where Q = 128is used for the indoor channel and Q = 256 for the 3GPP COST 207 TU channel. From the results, we can see that CS-based the interpolation method accomplishes interpolation based at the neighbourhood of the downsampled CIR points according to their DFT characteristics. Hence, the initial downsampled CIR should be obtained as accurately as possible. Moreover, the CS method is better than the methods using simple interpolation kernel such as linear function because it naturally recovers the CIR property from the sparse basis.

5.2 Two-stage MUIC

In the next part of our simulations, the grey-encoded 16-quadrature amplitude modulation is considered for the CP-CDMA system, which has N=64 symbols for each

block. The CP length is N/4 symbols. The multipath channel is composed of eight equivalent symbol-rate taps that are generated by the above indoor channel. For computational simplicity, two and four users, that is, P = 2 and 4, over independent channels are compared to show the efficiency of the two-stage MUIC algorithm. The spreading codes $c^{(u)}$, u = 1, 2, ..., P for P users are generated by Gold sequences of two different lengths $L_c = 15$ and 31. For all users, mutually non-overlapped preamble slots of two blocks, that is, 128 symbols, are assigned at the initial stage of transmission for channel and noise power estimation in the MUIC algorithm. The mandatory specification in IEEE 802.11 a/n/ac standards is referenced to setup our simulation parameters as listed in Table 1.

In our work, the MUI effect is reduced by the two-stage MUIC algorithm. Considering the case of two users, we compare the BER performance against the number of iteration for different L_c and SNR as shown in Fig. 5. The BER performance is better for a longer length of spreading code and larger SNR. No matter how long the length of spreading code is, at different SNR, the proposed iterative method almost achieves the steady-state performance with about two iterations. Therefore, in the next simulations we employ two iterations in the iterative algorithm.

The BER performance comparison of the new (proposed) CP-CDMA and conventional CP-CDMA for two users is

Table 1 Simulation parameters

Parameter	Value
modulation type	arev-encoded 16-QAM
FDE FFT size (N)	64
number of chips per symbol (L_c)	15 or 31
chip-rate length of CP (N_q)	240 or 496
samples per block for channel	128
estimation (Q)	
symbol rate	20 MHz
chip rate	300 MHz or 620 MHz
spreading code (c ^(u))	gold sequence
number of users (<i>P</i>)	2 or 4
multipath channel model	exponentially decaying
	power profile
chip-rate channel length (L _g)	120 or 248
average rms delay spread	82.18 ns

10 Lc=31 Lc=15 18dB 10 €20dB BER 10 -23dB 10 0 2 3 4 5 number of iteration

Fig. 5 BER against number of iterations used in the iterative MUI cancellation algorithm for two users



Fig. 6 BER comparison for two users with $L_c = 15$

shown in Figs. 6 and 7 with $L_c = 15$ and 31, respectively. The new despread-ahead CP-CDMA receiver without (w/o) using DFE [3] and the two-stage MUIC functions leads to the worst performance, and moreover, moving the code despreader ahead of the FDE results in severe MUI whether the DFE is performed or not. The BER performance of the proposed receiver is significantly improved with the help of the MUIC. From these two figures, a longer spreading code L_{c} =31 does further reduce MUI power (coincide with (23)), and thus, a better BER performance can be expected without doubt. Owing to the effect of the despread-ahead structure, more complicated MUI is introduced such that the conventional scheme has better BER performance than the proposed one even when the DFE and the MUIC are not employed. However, the two-stage MUIC algorithm effectively eliminates the MUI and gives a dramatic BER performance improvement. With the advantage of reducing the FFT size, the new receiver has little performance loss compared to the conventional one.

Another observation is that the performance improvement provided by the DFE is not as significant as that by the MUIC because the effect due to $\overline{W}_{k}^{(m)}(h, c, d)$ is actually far less than the MUI. As we simultaneously employ the DFE



Fig. 7 BER comparison for two users with $L_c = 31$



Fig. 8 BER comparison for four users with $L_c = 15$

and the MUIC, the new receiver has the BER performance almost close to the conventional one. Figs. 8 and 9 show the similar comparison for the case of four users. Although the proposed scheme shows a slightly worse performance than the conventional scheme because of more serious MUI effect, the new CP-CDMA receiver still benefits from great reduction of complexity on the FFT size.

5.3 Comment on complexity

In our framework, we assume that the channel is quasi-stationary in a frame and the CIR is estimated through two preamble blocks, that is, 128 symbols. Then, the estimated CIR is used for the despread-ahead CP-CDMA receiver along with the proposed two-stage MUIC algorithm. Hence, the complexity evaluation should be divided into two parts: initial CIR estimation and data detection.

5.3.1 Initial CIR estimation: Consider the symbol-rate FFT size is 64 and the spreading factor is 15. For the conventional CP-CDMA receiver, the FFT is employed at the chip rate and has size $N' = NL_c = 960$. The radix-2 FFT



Fig. 9 BER comparison for four users with $L_c = 31$

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complexity is $O(N'\log_2 N')$. However, the CIR estimation through the frequency-domain response requires N LS estimates, and the frequency-domain response is then transformed back to the time-domain CIR via N'-point IFFT. The overall complexity is $2O(N'\log_2 N') + N' = 19981$. For the CS-based algorithm, we require Q = 128frequency-domain measurements for the ℓ_1 norm minimisation. As we set the size of the transformed vector u as T=256, the complexity of a typical ℓ_1 minimisation scheme is generally O(kT + kQ) [36], where k is the required number of iterations for minimisation. In our cases, choosing k=20 leads to the overall complexity of ℓ_1 minimisation about 7680. The last work for the CS method is to transform the sparse vector u into CIR by $\hat{x} = \Psi u$, where Ψ is a $Q \times T$ matrix. Since *u* is sparse (about 50) non-zero values, transformed by DCT in our simulations), the CIR recovery complexity is about 128×50 . Then the overall complexity for the CS method is about 14080. Although in this case the complexity difference is not very significant, it increases as N' becomes larger. Besides, the LS channel estimates obtained from the N'-point FFT are very susceptible to noisy measurements while the CS method has better immunity to noisy measurements.

5.3.2 Data detection: In conventional CP-CDMA systems, the MUIC is also implemented at chip rate. The complexity due to the MUI canceller is accordingly the same for both the schemes. The main difference is in the operation of FFT. The FDE in the conventional scheme has the FFT size of L_c times more than that used in the proposed scheme. The complexity of the conventional FDE scheme is $O(N'\log_2N')$ while the proposed scheme is O (Mog_2N). For example, the conventional scheme leads to a ratio of about 24.8 more than the proposed scheme with $L_c = 15$ and a ratio of about 56.6 with $L_c = 31$.

Based on the above counting, the CS-based method provides good noise immunity ability for CIR estimation in addition to less complexity compared to the conventional chip-rate FFT/IFFT CIR estimation scheme. The despread-ahead structure also greatly reduces the requirements of FFT/IFFT computation.

6 Conclusion

In this paper, we provide the mathematical model of the despread-ahead CP-CDMA receiver with an iterative two-stage MUIC algorithm. From analytic and numerical results, this change does not cause failure to FDE detection even when we utilise Gold sequences as the spreading codes. The iterative two-stage MUIC algorithm only requires two iterations to reach a steady-state performance. In addition, we propose a novel CS-based channel estimation technique for the MUIC algorithm. As the CIR is compressible, the CS-based interpolation method can recover the CIR from the despreaded output without using complicated high-resolution interpolation kernels. The new CP-CDMA receiver shows a promising advantage of only requiring a small FFT size compared to the conventional CP-CDMA receiver.

7 References

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