Modified Decision Feedback Methods for OFDM Channel Tracking

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Abstract—**In an orthogonal frequency division multiplexing (OFDM) system, part of bandwidth is occupied by pilot symbols for time-varying channel estimation. In order to reduce the period of the training sequence while improving bandwidth efficiency, we study new time-domain channel tracking algorithms with a modified decision feedback loop. In company with the channel tracking algorithms, some decision error detectors and error concealment methods are proposed to reduce the error probability for avoiding decision error propagation. In our simulations, we compare the performance of the proposed decision error detectors and error concealment methods to show the efficiency of the proposed algorithms in time-varying channels.**

I. INTRODUCTION

The orthogonal frequency division multiplexing (OFDM) system plays an important role in beyond third generation communication services. It has been adopted in wireless networking and broadcasting applications (e.g. WLAN, DAB, and DVB-T, etc) and becomes a potential candidate of future standards. A main advantage of OFDM is its high bandwidth efficiency and simple equalization in a time-invariant channel. In contrast to traditional equalization, the OFDM system can avoid intersymbol interference (ISI) as the length of CP is longer than the delay spread of the channel impulse response. Moreover, the operations of FFT and IFFT make the equalization able to be implemented by dividing the subcarriers' channel gains. Hence, if the channel state information (CSI) is available, coherent demodulation can be performed easily by a single-tap equalizer.

Although some typical systems like IEEE 802.11a and IEEE 802.16d are designed for the application of a nearly static channel environment, the channel response may vary as the user changes his location with respect to the access point. In order to achieve a good receiver performance in a time-varying channel, the real-time channel tracking is required. Usually, channel estimation and tracking are accomplished by employing preambles and known pilots. The conventional pilot-aided algorithms have been studied in [1]. Reliable estimation results are obtained relying on sufficient pilots. However, the overhead due to pilots causes a considerable loss to the overall throughput. Recently, several studies have taken effort on reducing the overhead due to pilots. Channel estimation and tracking performed by decision feedback (DF) techniques are proposed [2]-[8]. Decision feedback algorithms have the good property that there is no communication overhead required. Typically, DF techniques for channel tracking are performed in the frequency domain [4], [5]. In this paper, we study time-domain channel tracking algorithms and the error-preventing structures. The time-domain least squares (LS) method could be found in MIMO applications [7], which has good tracking capability because it tracks fewer channel parameters than the frequency-domain method based on the same dimension of observations. Hence, estimation errors are spread over the complete frequency band, which means we can obtain more reliable channel estimation by using the time-domain method than the frequency-domain method.

However, DF techniques can cause the well-known error-propagation problem. In order to deal with the problem, we develop some error indicators to detect possible decision errors. We have observed that the decision errors usually result in an abrupt change on the value of the estimated channel response between neighboring subcarriers or come with a rather low channel gain. Hence, the error indicators designed for detecting channel abruption and low channel gain are studied. Simulation results show that new decision feedback methods achieve a better bit error rate (BER) performance than the methods without utilizing the decision error preventing structure.

II. DECISION FEEDBACK CHANNEL TRACKING METHODS

In a time-varying channel, we can exploit the preamble to initiate the channel estimation as in the acquisition mode and use pilots for the follow-up tracking mode. However, if there are no pilots designed for channel tracking, we should track it in the blind mode. In this paper, the blind algorithm can be referred to as the modified decision feedback algorithm.

A. Frequency-Domain LS with Decision Feedback

The frequency-domain decision feedback (FDF) solution is based on the LS criterion as follows:

$$\Psi_{\text{FDF}}(n) = \left\| \vec{Y}(n) - \hat{\bar{X}}(n)\vec{H}(n) \right\|^2 \quad (1)$$

where $\vec{Y}(n)$ is the $N \times 1$ FFT output vector at time *n*, $\hat{\bar{X}}(n) = \text{diag}[\hat{X}_0(n), \hat{X}_1(n), \cdots, \hat{X}_{N-1}(n)]$ where $\hat{X}_k(n)$ denotes the decision output on the *k*th subcarrier at time *n*, and $\vec{H}(n)$ is the $N \times 1$ frequency domain channel response vector. The LS solution to the channel estimation is

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$$\hat{\hat{H}}(n) = \arg\min_{H(n)} \Psi_{\text{FDF}}(n) = \hat{\hat{X}}^{-1}(n)\vec{Y}(n). \quad (2)$$

B. Time-Domain LS with Decision Feedback

The time-domain decision feedback (TDF) solution is based on the LS criterion in the time-domain:

$$\Psi_{\text{TDF}}(n) = \left\| \vec{Y}(n) - \hat{\vec{X}}(n) F_{N\times L} \vec{h}(n) \right\|^2 \quad (3)$$

where $\vec{h}(n)$ is the channel impulse response vector with elements $h_l(n), l = 0,1,...,L-1$, which are composed of L paths in a multipath channel and $F_{N\times L}$ is the $N\times L$ matrix transforming the channel impulse response into the frequency domain. The TDF solution to the channel impulse response is

$$\hat{\vec{h}}(n) = \arg\min_{h(n)} \Psi_{\text{TDF}}(n) = [F^H_{N\times L} \hat{\hat{X}}^H(n) \hat{\hat{X}}(n) F_{N\times L}]^{-1} [F^H_{N\times L} \hat{\hat{X}}^H(n) \vec{Y}(n)] \quad (4)$$

Notice that the TDF solution in (4) is more complicated than that of FDF in (2), but its performance is better because that only L taps are required to be estimated by calculating N subcarrier observations, where $N>L$.

C. The Kalman Filtering Algorithm with Decision Feedback

Suppose the transmitted signal after passing IDFT and adding cyclic prefix (CP) is $x_{CP}(n)$. The received signal at the receiver can be represented by the following vector form:

$$y(n) = \vec{x}_{CP}(n)\vec{h}(n) + v(n) \quad (5)$$

where

$$\vec{x}_{CP}(n) = [x_{CP}(n) x_{CP}(n-1) \cdots x_{CP}(n-L+1)],$$
$$\vec{h}(n) = [h_0(n) h_1(n) \cdots h_{L-1}(n)]^T,$$

and $v(n)$ is the zero-mean additive white Gaussian noise with variance $R(n)$. To establish the state estimation algorithm by applying the Kalman algorithm [9], we usually model the channel impulse response as a first order autoregressive (AR) process:

$$\vec{h}(n+1) = \Phi\vec{h}(n) + \vec{w}(n) \quad (6)$$

where $\Phi = \text{diag}[a_0 a_1 \cdots a_{L-1}]$, a_i is the AR(1) coefficient for the different L paths, and $\vec{w}(n)$ is the white Gaussian measurement noise vector with variance $Q(n)$. By using (5) and (6), the Kalman algorithm for the estimate of channel impulse response includes the following recursions:

$$P(n) = \Phi P(n-1)\Phi^T + Q(n) \quad (7)$$

$$K(n) = P(n)\vec{x}^T_{CP}(n)[\vec{x}_{CP}(n)P(t)\vec{x}^T_{CP}(n) + R^{-1}(n)] \quad (8)$$

$$e(n) = y(n) - \hat{y}(n) = y(n) - \vec{x}_{CP}(n)\hat{\vec{h}}(n-1) \quad (9)$$

$$\hat{\vec{h}}(n) = \Phi\hat{\vec{h}}(n-1) + K(n)e(n) \quad (10)$$

$$P(n+1) = [I - K(n)\vec{x}^T_{CP}(n)]P(n) \quad (11)$$

where $P(n)$ is known as the state prediction error covariance matrix, $P(n+1)$ is the state filtering error covariance matrix, and $K(n)$ is the Kalman gain. For the purpose of simplicity, we can set Φ as an identity matrix without significantly losing the performance.

The Kalman filtering algorithm used for channel tracking involves two stages. The first stage is to use the preamble as the initial training sequence of $x_{CP}(n)$. During the preamble period, the Kalman filter has to accomplish the channel estimation. The second stage is to transform the decision output signals into the time-domain signals through IDFT and then the CP signal is added. The resulted signal is used as the input of the Kalman filter in the tracking mode.

The variances of the process noise and the measurement noise can be approached by the following equations:

$$\overline{Q}(n) = \text{cov}[\hat{\vec{h}}(m) - \Phi\hat{\vec{h}}(m-1)] = \frac{1}{M}\sum_{m=n-M}^{n-1} \text{diag}\begin{bmatrix} \text{cov}[\hat{h}_0(m) - a_0\hat{h}_0(m-1)] \\ \text{cov}[\hat{h}_1(m) - a_1\hat{h}_1(m-1)] \\ \vdots \\ \text{cov}[\hat{h}_{L-1}(m) - a_{L-1}\hat{h}_{L-1}(m-1)] \end{bmatrix} \quad (12)$$

where M is the number of the average operation and

$$\overline{R}(n) = \frac{1}{M}\sum_{m=n-M}^{n-1} \text{cov}[y(m) - \vec{x}_{CP}(m)\hat{\vec{h}}(m)] \quad (13)$$

To reduce the computational complexity, the following recursive routines can be used to estimate the variances:

$$\hat{Q}(n) = \rho_1 \cdot \hat{Q}(n-1) + (1-\rho_1)\cdot\overline{Q}(n) \quad (14)$$

$$\hat{R}(n) = \rho_2 \cdot \hat{R}(n-1) + (1-\rho_2)\cdot\overline{R}(n) \quad (15)$$

where ρ_1 and ρ_2 are two forgetting factors and $0 \le \rho_1, \rho_2 < 1$.

III. Modified Decision Feedback Methods

Although decision feedback methods do not need pilots for channel tracking, the decision output possibly contains decision errors which can cause error propagation if these errors are used for channel estimation in the feedback loop. To deal with this problem, the tracking performance can be improved by applying a smoothing factor $\alpha, 0 < \alpha \le 1$, in the following recursive estimation equation [5]:

$$\hat{H}_{S,k}(n) = \alpha \cdot \hat{H}_{S,k}(n-1) + (1-\alpha)\cdot\hat{H}_k(n) \quad (16)$$

where $\hat{H}_k(n)$ denotes the output of the channel estimator on the kth subcarrier before the channel smoothing device and $\hat{H}_{S,k}(n)$ denotes the output of the smoothing device. $\hat{H}_k(n)$ can be obtained from (2), (4), or (10). Notice that (16) attenuates serious channel estimates caused by decision errors through the smoothing factor for a system originally proposed in a static channel to be applicable in a time-varying channel.

A. Detection of Decision Feedback Error

Consider the case that the channel response on a subcarrier changes slowly. Then, we can develop an indicator to detect an abrupt change on frequency response where a decision error possibly occurs. The following indicator is to calculate the normalized difference between the previously estimated channel and the present channel estimate:

$$T_k(n) = \begin{cases} 1, & \text{if } \dfrac{\left\| \hat{H}_k(n) \right| - \left| \hat{H}_{S,k}(n-1) \right\|}{\left| \hat{H}_{S,k}(n-1) \right|} \geq \gamma_1 \\ 0, & \text{otherwise} \end{cases} \qquad (17)$$

where $\gamma_1 > 0$ is a pre-determined threshold. If the indicator output is greater than a threshold, we decide that there is a decision error. However, if the channel response is in deep fading regions, we also decide an error by the following rule:

$$I_k(n) = \begin{cases} 1, & \text{if } \left| \hat{H}_k(n) \right| \leq \gamma_2 \\ 0, & \text{otherwise} \end{cases} \qquad (18)$$

where γ_2 is a pre-determined threshold. Based on the above error detection criterions, we can construct a composite error detector as follows:

$$D_k(n) = \begin{cases} 1, & \text{if } T_k(n) = 1 \text{ or } I_k(n) = 1 \\ 0, & \text{otherwise} \end{cases} \qquad (19)$$

B. Error Concealment

When an error location is decided, some strategy for error concealment is activated in order to reduce error propagation. We study two methods to compensate the unreliable estimate here. The first one is called "linear compensation" method based on three estimates near the unreliable estimate:

$$\hat{H}_{LN,k}(n) = \alpha \hat{H}_{C,k}(n-1) + (1-\alpha)[\hat{H}_{S,k-1}(n) + \hat{H}_{S,k+1}(n)]/2 \qquad (20)$$

$$\hat{H}_{C,k}(n) = [1 - D_k(n)] \cdot \hat{H}_{S,k}(n) + D_k(n) \hat{H}_{LN,k}(n) \qquad (21)$$

where the linear compensation, $\hat{H}_{LN,k}(n)$, is calculated by the channel estimate at time n-1 and the neighboring channel interpolation in the frequency domain. The concept is shown in Fig. 1

The second is called "extrapolation compensation" method based on four estimates near the unreliable estimates, which is represented as follows:

$$\hat{H}_{\Delta,k}(n) = \hat{H}_{C,k}(n-1) + [\hat{H}_{C,k}(n-1) - \hat{H}_{C,k}(n-\Delta)]/(\Delta - 1) \qquad (22)$$

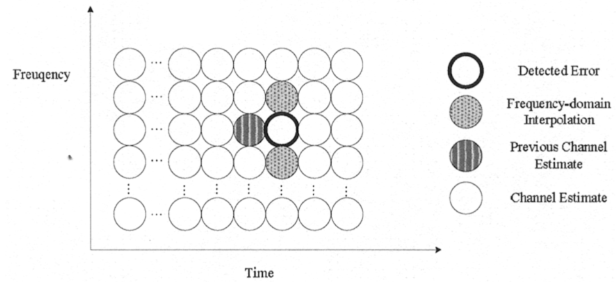

Fig. 1. Concept of the linear compensation.

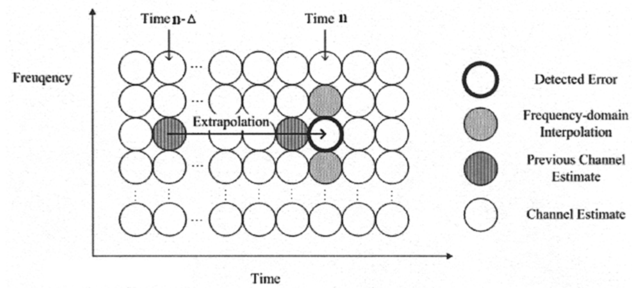

Fig. 2. Concept of the extrapolation compensation.

TABLE I. SYSTEM PARAMETERS

Parameters	Values
Center Frequency	2500 Mhz
Channel Bandwidth	2.5 Mhz
Length of FFT	256
Useful Symbol Duration (Tb)	91.4 ms
Guard Time (Tg=Tb/8)	11.4 ms
OFDM Symbol Duration (Tb+Tg)	102.9 ms
Frame Duration	5 ms
Number of OFDM Symbols	48
Time-varying Channel	16-tap Jakes

$$\hat{H}_{EX,k}(n) = \alpha \hat{H}_{\Delta,k}(n-1) + (1-\alpha)[\hat{H}_{S,k-1}(n) + \hat{H}_{S,k+1}(n)]/2 \qquad (23)$$

$$\hat{H}_{C,k}(n) = [1 - D_k(n)] \hat{H}_{S,k}(n) + D_k(n) \hat{H}_{EX,k}(n) \qquad (24)$$

where Δ is the number of delay for extrapolation. $\hat{H}_{EX,k}(n)$ is the weighted sum of the frequency-domain linear interpolation and the time-domain linear extrapolation, $\hat{H}_{\Delta,k}(n)$. The concept is depicted in Fig. 2.

IV. SIMULATIONS

In our simulations, we construct the system based on the 802.16d standard, the Wireless-MAN OFDM downlink, as listed in Tables 1. Each frame contains 48 OFDM data symbols and 2 preambles. For data transmission between successive training symbols, the receiver operates in the

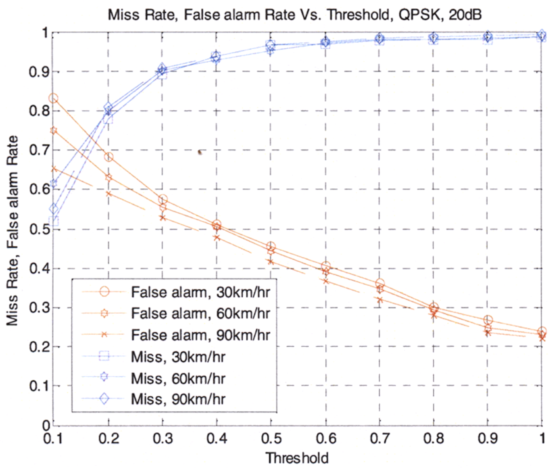

Fig. 3. Probability of false alarm and miss for $T_k(n)$.

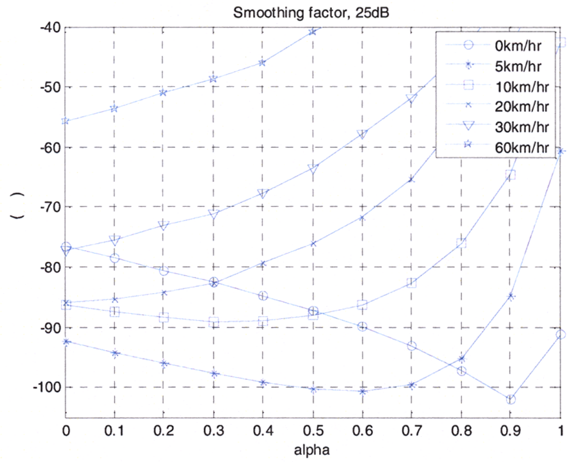

Fig. 5. MSE for different smoothing factors α at different speeds.

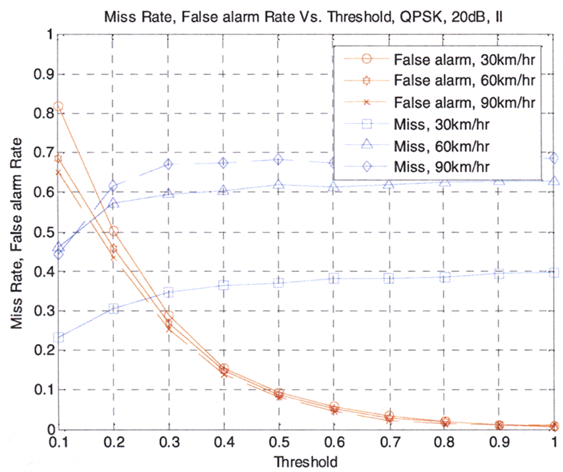

Fig. 4. Probability of false alarm and miss for $D_k(n)$.

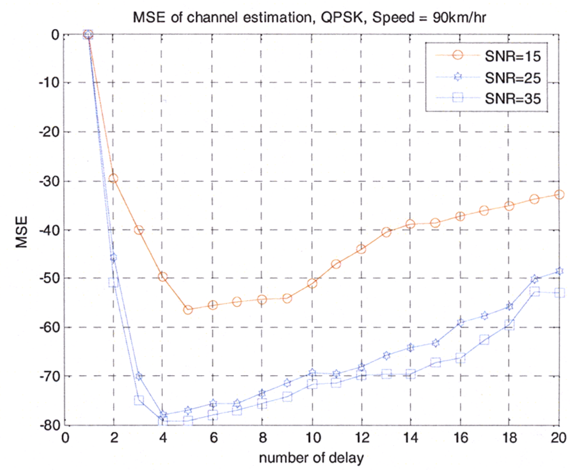

Fig. 6. MSE comparison for different numbers of delay at 90km/hr with extrapolation compensation.

channel tracking mode. The bit error rate (BER) curves are evaluated through averaging 100 OFDM fames, which include 5000 OFDM symbols for different time-varying channel realizations. Here, the Jakes model of 16 independent paths is adopted.

First, we compare the performance of the error detector based on the analysis of its false alarm and miss rate with respect to the threshold γ_1. The false alarm means there is no decision error, but the detector decides an error; while the miss means that there is a decision error, but the detector misses it. Both of them degrade the tracking performance. From Fig. 3 to Fig. 6, we use the time-domain Kalman DF algorithm for our simulation analysis.

Fig. 3 and Fig. 4 compare the false alarm rate and miss rate for different error detectors at different mobile speed. Notice that the probabilities of false alarm and miss by using $D_k(n)$ is lower than that by using $T_k(n)$, which means that $D_k(n)$ for the error detector is more reliable than $T_k(n)$.

Considering the Bayesian criterion, we can derive a risk function as follows:

$$F_{risk}(\gamma) = C_{FA} P_C P_{FA}(\gamma) + C_M P_E P_M(\gamma) \qquad (25)$$

where C_{FA} and C_M are the costs for false alarm and miss, P_C and P_E are probabilities of correct decision and false decision, and $P_{FA}(\gamma)$ and $P_M(\gamma)$ are the probabilities of false alarm and miss, respectively. The values of cost depend on the tradeoff between miss rate and false alarm rate. The optimum threshold can be obtained by minimizing the risk function calculated from the curves shown in Figs. 2 and 3. However, it is a tough work since P_C and P_E are related to the value of threshold.

Fig. 5 shows the relationship of channel estimation mean square error (MSE) and the smoothing factor α in (16) at SNR 25dB. From the simulation result, the smoothing method is not suitable for speed over 20km/hr in this system because the optimum smoothing factor becomes almost zero. Hence, the error compensation methods are necessary for fast time-varying channels.

In Fig. 6, we compare MSE of channel estimation for different SNR and different numbers of delay used in extrapolation compensation. Notice that in these cases, small delay numbers, 1 and 2, are worse than others. The reason is when the error detector shows a warning signal, the previous 1 or 2 symbols of channel estimate is also unreliable. The problem can be better remedied by exploiting the advance 4 or 5 symbols of channel estimate. It is shown that when we adopt the delay of 5 symbols, we have the almost minimum MSE performance at SNR=15dB, 25dB, 35dB when the mobile speed is 90km/hr.

In Figs. 7 and 8, we compare the BER performance of three channel estimators and two error compensation methods introduced in section II and III. The channel estimators include FDF, TDF, and the time-domain Kalman filtering (TKF) algorithms and the error compensation methods are "Linear" (Ln) and "Extrapolation" (Ex). Fig. 7 and Fig. 8 show results at the speed of 30 km/hr and 90 km/hr, respectively. Here, we set the parameters as $\alpha = 0.3$, $\gamma_1 = 0.8$, and $\gamma_2 = 0.2$. The FDF performs worse than others because TDF and TKF estimate the channel impulse response of only 16 taps while FDF estimates the frequency response of 256 subcarriers. However, TDF and TKF require higher computational complexity to achieve better results compared to FDF. The Ex compensation method shows obvious performance improvement over the Ln method for both TDF and TKF algorithms.

V. Conclusion

This paper introduces low overhead channel tracking algorithms for the OFDM system in a time-varying channel. We show that the time-domain decision feedback algorithm has better performance than the frequency-domain decision feedback algorithm. The proposed error compensation structure substantially improves the performance of decision feedback channel tracking algorithms.

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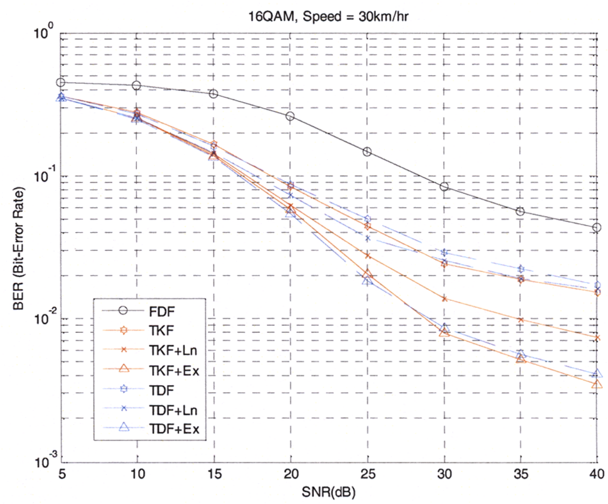

Fig. 7. BER performance comparison of 16-QAM for FDF, TDF, and TKF algorithms with 4% training symbols at 30 km/hr.

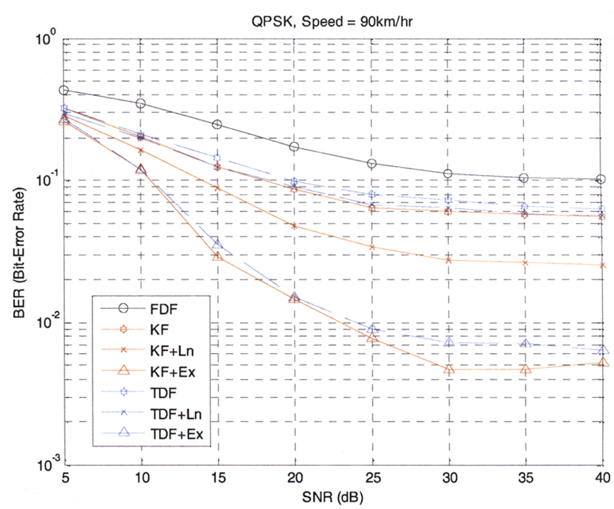

Fig. 8. BER performance comparison of QPSK for FDF, TDF, and TKF algorithms with 4% training symbols at 90 km/hr.