Least Squares/Maximum Likelihood Methods for the Decision-Aided GFSK Receiver

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***Abstract*—Gaussian frequency shift keying (GFSK) modulation is widely used in low data-rate wireless audio/video transmission and personal communication standards like Bluetooth. In this letter, least squares (LS) and maximum likelihood (ML) methods are exploited for decision-aided carrier recovery in a GFSK receiver. Through the analysis of estimation error variance, we show that the LS/ML carrier frequency offset estimation method outperforms the conventional method using the DFT approach. Besides, the new method is also effective to deal with non-DC-free data sources. Simulation results show that the new decision-aided GFSK receiver achieves better results than the feedforward receivers with DC-free and non-DC-free sources.**

***Index Terms*—Bluetooth, carrier frequency offset, decision-aided method, GFSK, maximum likelihood.**

I. Introduction

GAUSSIAN frequency shift keying (GFSK) is a kind of continuous phase frequency shift keying (CPFSK) modulation technique originated from FSK which is a well-known power-efficiency modulation scheme. However, the bandwidth requirement of FSK significantly increases as the number of modulating symbols increases. In modern low data-rate applications, GFSK modulation employs the Gaussian function as a pulse shaping filter to reduce transmission bandwidth.

An important issue of implementing the GFSK demodulator is synchronization control. The timing correction problem can be referred to the well-known work proposed by Gardner. To cope with carrier synchronization, some classic carrier frequency offset (CFO) estimation methods [1]–[3] developed for minimum shift keying (MSK) type modulation can be considered. In [1], CFO can be obtained from the angle of the autocorrelation function of received signals. Morelli and Mengali [2] modified the 2-power autocorrelation method [3] used in MSK for Gaussian MSK (GMSK) modulation. In [4], although CFO can be directly calculated from received signals and re-modulated transmitted symbols, the channel response and the training sequence are required to be known in advance. Besides, some algorithms using stochastic gradient [5], maximum likelihood estimation [6], and discrete-time Fourier transform [7] also can be found in the literature.

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Most of the previous methods are feedforward processing of received signals and the transmitted binary data are assumed to have equal probability. If the binary symbols are equally probable, the GFSK signal has zero direct current (DC) value which is called "DC-free". However, the assumption of equal probability may be not true over a finite length of interval or for a specific data source which can be treated as "non-DC-free." When the transmitted data are non-DC-free, the bit error rate (BER) performance may be degraded by using previous carrier synchronization methods based on the DC-free assumption. In this letter, we derived a decision-aided carrier recovery method for the GFSK receiver which can effectively deal with non-DC-free data. The GFSK signal is demodulated by applying a discrete-time differential phase detector where the CFO results in a DC level bias to the detector output. Then, least-squares (LS)/maximum likelihood (ML) methods are developed to estimate CFO. With the help of the decision output, a recursive equation can be realized for the purposes of low complexity and high feasibility. In comparison to the DFT method presented in [7], the proposed method does not only solve the non-DC-free problem efficiently, but also has a better estimation performance in the DC-free case. Simulation results show that the proposed method can work at low signal to noise ratio (SNR) even though the decision-aided output is used.

II. GFSK Modulation

The functional block of a typical GFSK modulator is plotted in Fig. 1(a). The input signal $x(t)$ is first filtered by the Gaussian filter $h(t)$ and then modulated by the frequency generator consisting of an integrator and a quadrature frequency synthesizer. The product of the 3-dB filter bandwidth B and the symbol period T is defined as BT. The impulse response of the Gaussian filter can be expressed as

$$h(t) = \frac{1}{\sqrt{2\pi}\sigma T} \exp\left(\frac{-t^2}{2\sigma^2 T^2}\right) \quad (1)$$

where $\sigma = \sqrt{\ln(2)}/2\pi BT$. Let the input signal $x(t)$ be the random bipolar pulse for binary transmission, and

$$x(t) = \sum_{n=-\infty}^{\infty} x[n] \cdot \Pi((t-nT)/T) \quad (2)$$

where $x[n]$ are the discrete-time samples of $x(t)$, $x[n] \in \{+1, -1\}$, and $\Pi(t/T)$ is the rectangular pulse function with a value $1/T$ for $|t| < T/2$ only. The pulse shaping function for the binary data $x[n]$ becomes $g(t) = h(t) * \Pi(t/T)$, where the notation * denotes the linear convolution operation. By

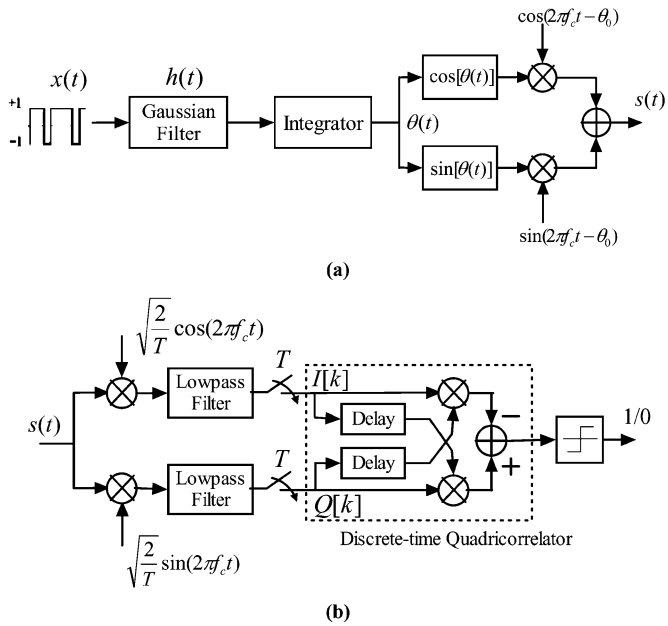

Fig. 1. Functional blocks of a typical GFSK transceiver: (a) modulator (b) the discrete-time quadricorrelator.

mathematical derivation, the pulse shaping function can be obtained as

$$g(t) = \frac{1}{T}\left[Q\left(\frac{t-T/2}{\sigma T}\right) - Q\left(\frac{t+T/2}{\sigma T}\right)\right] \tag{3}$$

where $Q(\cdot)$ is the Q-function and is defined as $Q(t) = \int_t^{\infty} e^{-\tau^2/2}/\sqrt{2\pi}d\tau$. The continuous phase generated by the frequency modulation is

$$\theta(t) = 2\pi h \int_{-\infty}^{t} \sum_{n=-\infty}^{\infty} x[n]g(\tau - nT)d\tau \tag{4}$$

where h is the modulation index. We may observe that the length of the function $g(t)$ determines how many symbols are involved in the calculation of the phase. For $BT = 0.5$, the general choice for the length of the Gaussian filter $h(t)$ is ± 2 symbol spans. The GFSK modulation signal with unit power at the center frequency f_c is

$$s(t) = \sqrt{\frac{2}{T}}\cos[2\pi f_c t - \theta(t) - \theta_0] \tag{5}$$

where θ_0 is an initial phase offset.

III. LS/ML Methods for Decision-Aided Carrier Recovery

A. Mathematical Model for GFSK Receiver

In (4), we let $\tilde{x}(t) = \sum_{k=-\infty}^{\infty} x[k]g(t-kT)$. Since the discrete-time version $\tilde{x}(kT)$ of $\tilde{x}(t)$ is correlated to $x[k]$, we can see that $\tilde{x}(t)$ is the intersymbol interference (ISI) version of $x(t)$. Suppose the CFO f_δ exists between the transmitter and the receiver. From (5) and Fig. 1(b), the received baseband signals of in-phase (I) and quadrature (Q) components at time instants k and $k-1$ can be found as

$$I[k] = I(t)\,|_{t=kT} = \cos(\alpha) + w_I[k] \tag{6}$$
$$I[k-1] = I(t)\,|_{t=(k-1)T} = \cos(\beta) + w_I[k-1] \tag{7}$$
$$Q[k] = Q(t)\,|_{t=kT} = \sin(\alpha) + w_Q[k] \tag{8}$$
$$Q[k-1] = I(t)\,|_{t=(k-1)T} = \sin(\beta) + w_Q[k-1] \tag{9}$$

where $w_I[k], w_I[k-1], w_Q[k]$, and $w_Q[k-1]$ are modeled as additive white Gaussian noises (AWGN) and

$$\alpha = 2\pi\left[h\int_{-\infty}^{kT}\tilde{x}(\tau)d\tau + f_\delta kT\right] + \theta_0 \tag{10}$$

$$\beta = 2\pi\left[h\int_{-\infty}^{(k-1)T}\tilde{x}(\tau)d\tau + f_\delta (k-1)T\right] + \theta_0. \tag{11}$$

The output of the quaricorrelator is

$$\begin{aligned} I[k-1]Q[k] &- I[k]Q[k-1] \\ &= \sin\left\{2\pi\left[h\int_{(k-1)T}^{kT}\tilde{x}(\tau)d\tau + f_\delta T\right]\right\} + w'[k] \\ &= \sin\{2\pi T[h\tilde{x}(kT) + f_\delta]\} + w'[k] + w_s[k] \end{aligned} \tag{12}$$

where $w'[k] = w_I[k-1]w_Q[k] - w_I[k]w_Q[k-1]$ and $w_s[k]$ accounts for the integration approximation error. In (12), the integration of $\tilde{x}(\tau)$ between $(k-1)T$ and kT is approximated by $\tilde{x}(kT)T$. Besides, by using the approximation $\sin\phi \approx \phi$ for a small value of ϕ, we have that if $f_\delta T$ is small enough, the output obtained from (12) after dropping the factor $2\pi T$ becomes

$$\hat{x}[k] \approx h\tilde{x}[k] + f_\delta + w[k] \tag{13}$$

where $\tilde{x}[k]$ denotes $\tilde{x}(kT)$ and $w[k] = (w'[k] + w_s[k])/2\pi T$. Note that from (13), CFO causes a DC level drift at the output and thus increases the symbol decision error rate.

B. Estimation of Carrier Frequency Offset

Assume that transmitted binary symbols are equally probable. The N-point discrete-time Fourier transform (DFT) $\Psi[k]$ of $\hat{x}[n]$ is defined as

$$\Psi[k] = \sum_{n=0}^{N-1}\hat{x}[n]\exp\left\{-j\frac{2\pi kn}{N}\right\}, \quad 0 \le k \le N-1. \tag{14}$$

It was proposed that if the sampled function $g[n]$ of $g(t)$ is even and real with zero dc, then the frequency offset f_δ can be estimated by [7]

$$\hat{f}_\delta^{\mathrm{DFT}}[n] = \frac{1}{N}\Psi[0]. \tag{15}$$

Consider the case that the assumption of equal probability is not true and then (15) does not hold for a non-DC-free source. Note that although the noise $w[k]$ is not a Gaussian process, it is zero-mean. However, we can choose the value f_δ that makes $h\tilde{x}[k] + f_\delta$ closest to its perturbed version $\hat{x}[k]$. In this case, the closeness can be measured by the LS error criterion based on the N nearest observations:

$$J(f_\delta) = \sum_{k=n-N+1}^{n} (\hat{x}[k] - h\tilde{x}[k] - f_\delta)^2. \quad (16)$$

The LS method estimates f_δ by means of minimizing $J(f_\delta)$. Since $J(f_\delta)$ is a quadratic function of f_δ, we can solve $\partial J(f_\delta)/\partial f_\delta = 0$ to find f_δ. The LS estimation yields

$$\hat{f}_\delta^{\mathrm{LS}}[n] = \frac{1}{N} \sum_{k=n-N+1}^{n} (\hat{x}[k] - h\tilde{x}[k]). \quad (17)$$

Suppose the number of observations is large enough. By the Central Limit theorem, the noise term in (13) can be approximated as the Gaussian process with variance σ_w^2. Then, the ML estimation method can be also used to find the same estimate of f_δ as (17) based on the following joint log-likelihood function obtained from (13),

$$\Lambda(f_\delta) = \frac{-1}{2\sigma_w^2} \sum_{k=n-N+1}^{n} (\hat{x}[k] - h\tilde{x}[k] - f_\delta)^2. \quad (18)$$

The Cramer–Rao lower bound (CRLB) for the estimation error variance can be found as

$$E\{|\hat{f}_\delta^{\mathrm{LS}} - f_\delta|^2\} \geq \sigma_w^2/N. \quad (19)$$

C. Recursive Decision-Aided Implementation

To solve (17), $\tilde{x}[k]$ shall be reconstructed at the receiver. In order to reduce complexity, we use the hard decision output $x_d[k]$ of $\hat{x}[k]$ to replace $h\tilde{x}[k]$, where the difference between $x_d[k]$ and $h\tilde{x}[k]$ can be treated as an additive zero-mean noise to the right hand side of (13). Hence, the decision-aided LS/ML approach can be realized at a reasonable low symbol error rate. The following recursive equation can be efficient to implement (17):

$$\hat{f}_\delta[n] = \hat{f}_\delta[n-1] + K(\hat{x}[n] - x_d[n]) \quad (20)$$

where K is the parameter determining estimation error variance and convergence rate. In (20), the initial value $\hat{f}_\delta[0]$ can be simply set to zero without influencing the convergence behavior from our simulations. As the CFO signal f_δ is estimated, a second-order phase-locked loop (PLL) is used for carrier recovery. Note that the frequency offset due to the mismatch of local oscillators in general remains to an acceptable level such that the DC bias will not result in lots of decision errors at low SNR and the proposed carrier recovery algorithm can be applied.

D. Comparison of Estimation Error Variance

Even with non-DC-free data sources and assuming perfect decision feedback, we can show that the LS estimation scheme removes the effect of data and thus is more effective than (15). By ignoring the noise term, the estimation error variance of $\hat{f}_\delta^{\mathrm{DFT}}[n]$ for DC-free data can be calculated as

$$\begin{aligned}\mathrm{Var}\left\{\hat{f}_\delta^{\mathrm{DFT}}[n]\right\} &= E\left[\left(\frac{1}{N}\sum_{k=n-N+1}^{n}\hat{x}[k] - f_\delta\right)^2\right] \\ &= \frac{h^2}{N^2}E[\tilde{\mathbf{x}}^T\tilde{\mathbf{x}}]\end{aligned} \quad (21)$$

where $E[\cdot]$ denotes the expectation operation and $\tilde{\mathbf{x}} = [\tilde{x}[n-N+1]\tilde{x}[n-N+2]\cdots\tilde{x}[n]]^T$. By ignoring the decision error at a reasonable transmission error rate, the estimation error variance of (17) is

$$\begin{aligned}&\mathrm{Var}\left\{\hat{f}_\delta^{\mathrm{LS}}[n]\right\} \\ &= E\left[\left(\frac{1}{N}\sum_{k=n-N+1}^{n}(\hat{x}[k] - x_d[k]) - f_\delta\right)^2\right] \\ &= \frac{1}{N^2}\left(h^2E[\tilde{\mathbf{x}}^T\tilde{\mathbf{x}}] + E\left[\mathbf{x}_d^T\mathbf{x}_d\right] - hE[\tilde{\mathbf{x}}^T\mathbf{x}_d] \right. \\ &\quad \left. - hE\left[\mathbf{x}_d^T\tilde{\mathbf{x}}\right]\right)\end{aligned} \quad (22)$$

where $\mathbf{x}_d$ is $[x_d[n-N+1]x_d[n-N+2]\cdots x_d[n]]^T$. Denote the difference between $\mathbf{x}_d$ and $h\tilde{\mathbf{x}}$ as $\mathbf{x}_e$, that is, $\mathbf{x}_d = h\tilde{\mathbf{x}} + \mathbf{x}_e$. Neglect the slight correlation between the zero-mean noise $\mathbf{x}_e$ and $\tilde{\mathbf{x}}$. At a reasonable low decision error rate, (22) becomes

$$\mathrm{Var}\left\{\hat{f}_\delta^{\mathrm{LS}}[n]\right\} = \frac{h^2}{N^2}E\left[\mathbf{x}_e^T\mathbf{x}_e\right]. \quad (23)$$

Obviously, we have that $\mathrm{Var}\{\hat{f}_\delta^{\mathrm{DFT}}[n]\} > \mathrm{Var}\{\hat{f}_\delta^{\mathrm{LS}}[n]\}$. This represents a better estimation performance with the LS technique.

IV. System Simulations

In our simulation, the GFSK parameter for BT is 0.5, modulation index h is 0.5, and the filter length to be calculated is ± 2 symbols span. To improve the resolution of the filter response implemented with discrete-time processing, the modulation signal is oversampled at a ratio of 8. The symbol rate is 3 M symbols/s and thus, the operation rate is 24 MHz at the transmitter. The architecture of the GFSK receiver is depicted in Fig. 2. The immediate frequency (IF) of the received GFSK signal is set to be 8 MHz. To simplify the baseband receiver, the sample rate of the analog-to-digital (A/D) converter at the receiver is chosen as 24 MHz. The digital IF signal is then converted to the baseband signal at the sample rate 12 MHz after a digital down-converter (DDC) where a 21-tap FIR lowpass filter is included with parameters: passband at 2 MHz, stopband at 5 MHz, and stopband attenuation of 40 dB. The timing recovery is implemented by a cubic Lagrange interpolation filter with the Gardner timing error estimation algorithm and the carrier recovery is implemented by the proposed algorithm at the symbol rate.

The determination of K is a tradeoff between steady-state performance and the convergence rate. Here, we choose $K =$

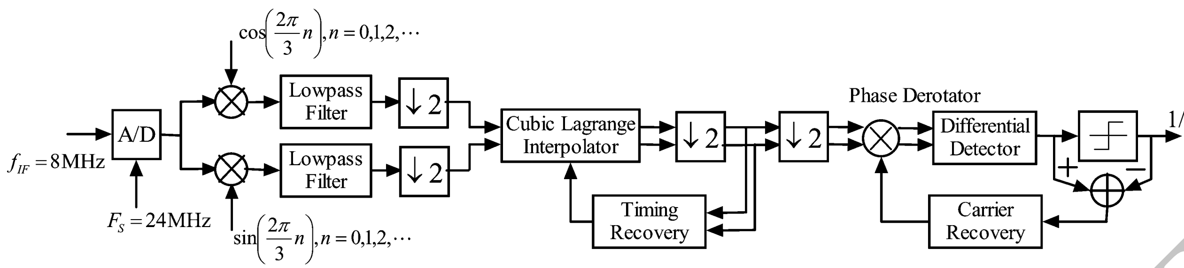

Fig. 2. System architecture of the GFSK receiver.

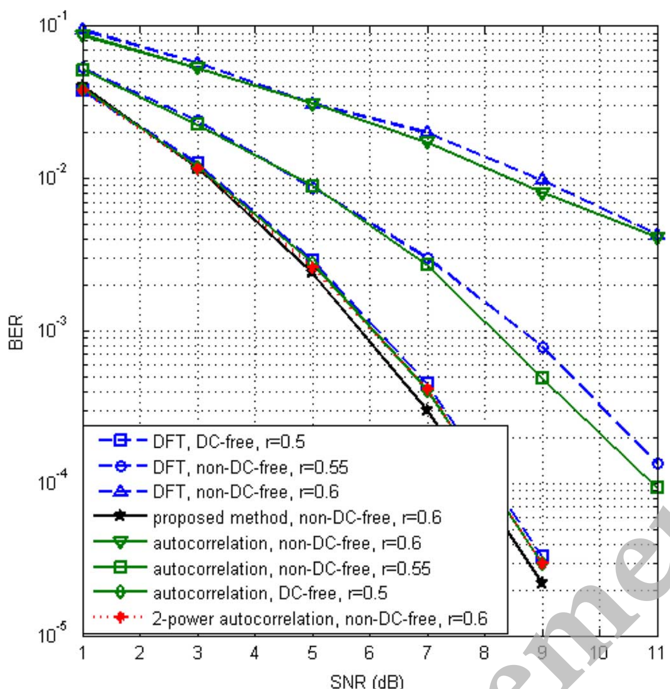

Fig. 3. BER performance comparison of the proposed method, the average method, and the autocorrelation method with AWGN and 100 KHz CFO for DC-free and non-DC-free data sources.

2^{-6}. In Fig. 3, we compare the proposed method with the autocorrelation method [1], the 2-power autocorrelation method [1], [2], and the DFT method [7] with 100 KHz CFO. The non-DC-free data sources are generated with the number of binary "1" to the number of binary "0" ratio $r = 0.5, 0.55$, and 0.6. Note that the autocorrelation method requires transmitted symbols to be equally probable and the 2-power auto- correlation method costs more computations. The performance of the average and autocorrelation methods is significantly degraded with non-DC-free data. The proposed method for a non-DC-free data source has almost the same performance for a DC-free data source. Note that although the proposed method is the feedback approach, its performance has no significant degradation and is close to that obtained by the feedforward approach for DC-free data sources at low SNR.

V. Conclusion

The LS/ML estimation is applied to a new decision-aided carrier recovery method for the GFSK receiver. The CFO can be obtained from the output of a differential phase detector through the presented mathematical model. The new CFO estimator eliminates the influence of decision data, and thus, outperforms the DFT approach in estimation error variance. By remedying the assumption of equally probable binary data, the new method is robust to non-DC-free data sources as compared to the feedforward approaches.

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