Spatial-Division Multiplexing MIMO Detection Based on a Modified Layered OSIC Scheme

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Abstract-Spatial-division multiplexing (SDM) provides very high spectral efficiency in multiple-input multiple-output (MIMO) systems. A well-known SDM-MIMO wireless system is vertical Bell Labs layered space-time (V-BLAST) which exhibits a good tradeoff between performance and complexity. Although maximum likelihood detection (MLD) has the optimal performance, its complexity is too high to practice such that some alternatives have been studied. The ordered successive interference cancellation (OSIC) algorithm was proposed for the advantage of high feasibility, however, there is a significant performance gap between MLD and OSIC. Here, we propose a modified layered OSIC algorithm to improve symbol detection in ill-conditioned layers with lower complexity compared to exhaustive search methods. To reduce the number of calculating matrix inversion for optimal ordering, we introduce a modified parallel interference cancellation method with precancellation and postcancellation to replace part of successive interference cancellation, based on evaluating the post-detection signal to noise ratio for each layer. Complexity analysis shows that the proposed algorithm saves about 65% operation of matrix inversion compared to a near-optimal improved layered OSIC algorithm while maintaining the similar bit error rate performance as shown in numerical results.

Index Terms—Spatial-division multiplexing, multiple-input multiple-output (MIMO), interference cancellation, vertical Bell Labs layered space-time (V-BLAST), maximum likelihood detection.

I. INTRODUCTION

MULTIPLE-INPUT multiple-output (MIMO) is a wireless communication technology that uses multiple transmit and receive antennas for data transmission at the same time, in order to massively promote channel capacity. Compared to a traditional single-input single-output (SISO) system, a MIMO system with N_t transmit and N_r receive antennas can increase its channel capacity as $\min(N_t, N_r)$ increases without additional context of transmission power and bandwidth [1]. IIMO technology can be broadly divided into two classes: spatial diversity and spatial multiplexing (SM). The former such as the space-time block code (STBC) uses multiple transmission paths provided by multiple transmit

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and receive antennas to combat the channel fading effect for the purpose of enhancing signal reliability [2], [3]. The latter transmits serial data stream simultaneously in different parallel sequences over multiple antennas, and then solves the sequences separately at the receiving end. By way of this procedure, it is possible to regard the transmission as a group of parallel spatial channels to serve high transmission rate. The vertical Bell Laboratories layered space-time system (V-BLAST) is the most representative technology [4], [5].

Some algorithms have been studied to improve signal detection in an SM-MIMO system. For instance, linear detection methods such as zero forcing (ZF) and minimum mean square error (MMSE) are very simple in structure, but optimal performance is not guaranteed. Maximum likelihood detection (MLD) can reach the optimal result with the lowest bit error rate (BER), but the complexity grows exponentially with the size of modulation dimension and the number of antennas [6], [77], which consequently impedes practical implementation. In order to solve this problem, one alternative is to develop near-optimal detection methods under the main demand of reducing the order of complexity to be reasonably feasible while approaching the MLD performance. In the literature, we can find near optimum detection [8], sphere decoding (SD) [9]-[11], K-best SD [12], QR decomposition and Malgorithm (QRD-M) [13], [14], successive and ordered successive interference cancellation (OSIC) [5], [15], [16], parallel interference cancellation (PIC) [17], etc. In these algorithms, the OSIC approach has drawn a lot of interests because of lower complexity, however, its performance is significantly worse than the MLD. Recently, some OSIC based algorithms were proposed for improved complexity or performance. Since the main complexity in the OSIC algorithm is to compute pseudoinverse, low-complexity methods mainly focus on simplifying matrix operation [18], [19]. To improve the performance of SIC detection, the ML method has been considered to combine with the OSIC algorithm [20] but the problem of the ML detection length and resultant performance were not distinctly addressed. In [21], the performance drawback due to the ill-conditioned sub-channel in the OSIC algorithm was mentioned and using exhaustive search for the worst layer was proposed to enhance the OSIC algorithm. Unfortunately, it requires to be extended for deploying a large number of antennas, and hence, also experiences the complexity issue.

Owing to the deficiencies of previously proposed methods we study a new modified layered OSIC algorithm. The reliability of weak layers is improved through exploring possible transmitted symbols from active search branches which are chosen by a pre-determined search region in constellation.

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Some low-possibility branches are purged away to reduce the number of searching. A modified OSIC algorithm that takes advantages of parallel and successive interference cancellation is also proposed in order to reduce the number of employing pseudoinverse while maintain a satisfying performance. Compared with the conventional OSIC algorithm, the new algorithm further propels the BER performance toward the MLD's result with better feasibility. From the results of complexity analysis, the new algorithm achieves a very close performance to that obtained by extending the improved layered (IL) OSIC method [21] which almost performs as well as the MLD, along with a saving of about 65% matrix inversion.

Recently, iterative ("Turbo") processing techniques have received considerable attention for multiuser interference suppression in code-division multiple-access (CDMA) systems [22], [23] and then the list-type detection algorithms also continued their development to improve detection performance with soft interference cancellation and decoding for the V-BLAST system [24]–[28]. In [24], the error propagation effect due to successive detection in the V-BLAST was taken into account for better performance when finding the MMSE solution and deriving the iterative decoding scheme. The scheme in [25] derived the symbol estimator by minimizing the interference plus noise power, given a priori probabilities for undetected layers and a posteriori probabilities for past detected layers. Lamare et. al's scheme [23] proposed an iterative successive and parallel interference cancellation structure in a decision feedback receiver along with a nearoptimal low-complexity user ordering algorithm. To combat error propagation in the decision feedback loop, the multiple feedback SIC method [26] was then developed for a multiuser MIMO system. Our new algorithm focuses on a complexitypromising scheme for symbol search on ill-conditioned layers and partially parallel layered detection to pursue a suboptimum performance which is very close to the extended IL OSIC's result. And, it can be also tailored to a list-type detector for a considerable performance gain.

The rest of this paper is organized as follows. Section II introduces the system model and the soft-output detection method used in this paper. Section III detailedly describes the proposed modified layered OSIC method and also summarizes the overall algorithm. Section IV gives performance discussion about the search branch reduction method and the influence of performing symbol search on ill-conditioned layers. In Section V, the complexity issue and BER performance are numerically evaluated. The conclusion to this work comes up in Section VI.

II. SYSTEM MODEL

As depicted in Fig. 1, consider an $N_t \times N_r$ MIMO system where N_t is the number of transmit antennas and N_r the number of receive antennas. The binary source is first passed through the Gray-encoded QAM mapper to generate a complex signal vector \mathbf{x} in the 2^M -QAM signal constellation, where M is the number of modulated bits in a symbol. Then, the Spatial Stream Parser divides the QAM signals into N_t layers. Taking into consideration the assignment method of signals in layers, the V-BLAST structure is assumed in our framework. Suppose perfect synchronization and channel



Fig. 1: Transmitter and receiver in a V-BLAST MIMO system.

state information are obtained at the receiver. We denote by $s^{(i)}(k)$, $i = 1, 2, \dots, N_t$ the N_t complex input signals at time k and $\mathbf{s}(k) = [s^{(1)}(k), s^{(2)}(k), \dots, s^{(N_t)}(k)]^T$ the $N_t \times 1$ transmit symbol vector. The $N_r \times 1$ complex vector $\mathbf{y}(k) = [y^{(1)}(k), y^{(2)}(k), \dots, y^{(N_r)}(k)]^T$ denotes the receive symbol vector. Assuming that the MIMO channels are complex Rayleigh fading, we have

$$\mathbf{y}(k) = \mathbf{H}(k)\mathbf{s}(k) + \mathbf{n}(k), \tag{1}$$

where $[\mathbf{H}(k)]_{ij} = h_{ij}(k), 1 \le i \le N_r, 1 \le j \le N_t$ and $\mathbf{n}(k) = [n_1(k) \ n_2(k), \cdots, n_{N_r}(k)]^T$. Here, $h_{ij}(k)$ represents the channel coefficient between the *j*th transmit antenna and the *i*th receive antenna, which can be treated as a complex Gaussian random variable. Those elements $n_j(k), j = 1, 2, \cdots, N_r$ in noise vector $\mathbf{n}(k)$ are mutually independent and identically distributed complex Gaussian.

For simplicity, we omit the time index k in the following content. The detection performance can be improved with a soft-input-soft-output channel decoder for the soft detection output. By (1), the noise vector can be written as $\mathbf{n} = \mathbf{y} - \mathbf{Hs}$. Let **n** be zero-mean with covariance matrix $\sigma_n^2 \mathbf{I}_{N_r}$ where \mathbf{I}_{N_r} denotes an identity matrix of size N_r . Its probability density function becomes

$$f_{\mathbf{N}}(\mathbf{n}) = \frac{1}{(2\pi)^{N_r/2} \sigma_n^{N_r}} \exp\left(-\frac{1}{2\sigma_n^2} \|\mathbf{y} - \mathbf{Hs}\|^2\right)$$
$$= f_{\mathbf{Y}}(\mathbf{y}|\mathbf{s}).$$
(2)

Let us move on to evaluate the bit-level log-likelihood ratio (LLR) value. Denote $b_{l,i}$ the *l*th bit for the *i*th transmit antenna. The a priori probability of transmitting "0" or "1" is equal as well, i.e., $p(b_{l,i} = 1) = p(b_{l,i} = 0) = 1/2$. We can express

the bit-level LLR as follows:

$$LLR(b_{l,i}|\mathbf{y}) \triangleq \ln \frac{p(b_{l,i} = 1|\mathbf{y})}{p(b_{l,i} = 0|\mathbf{y})}$$
$$= \ln \frac{f_{\mathbf{Y}}(\mathbf{y}|b_{l,i} = 1)}{f_{\mathbf{Y}}(\mathbf{y}|b_{l,i} = 0)}$$
$$= \ln \frac{\sum_{\mathbf{s}\in\mathbf{S}_{l,i}^+} \exp\left(-\frac{1}{2\sigma_n^2} \|\mathbf{y} - \mathbf{Hs}\|^2\right)}{\sum_{\mathbf{s}\in\mathbf{S}_{l,i}^-} \exp\left(-\frac{1}{2\sigma_n^2} \|\mathbf{y} - \mathbf{Hs}\|^2\right)}, \quad (3)$$

where $\mathbf{S}_{l,i}^+$ and $\mathbf{S}_{l,i}^-$ represent the collections of "1" and "0" for the *l*th bit on antenna *i*, respectively. Then, using the approximation to the log function, $\log (e^{X_1} + e^{X_2} + \dots + e^{X_n}) \approx \max X_i$, we can simplify the result in (3) as

$$LLR(b_{l,i}|\mathbf{y})$$

$$\approx \max_{\mathbf{s}\in\mathbf{S}_{l,i}^{+}} \left(-\frac{1}{2\sigma_{n}^{2}} \|\mathbf{y} - \mathbf{Hs}\|^{2} \right) - \max_{\mathbf{s}\in\mathbf{S}_{l,i}^{-}} \left(-\frac{1}{2\sigma_{n}^{2}} \|\mathbf{y} - \mathbf{Hs}\|^{2} \right)$$

$$= \frac{1}{2\sigma_{n}^{2}} \min_{\mathbf{s}\in\mathbf{S}_{l,i}^{-}} \|\mathbf{y} - \mathbf{Hs}\|^{2} - \frac{1}{2\sigma_{n}^{2}} \min_{\mathbf{s}\in\mathbf{S}_{l,i}^{+}} \|\mathbf{y} - \mathbf{Hs}\|^{2}.$$
(4)

The main problem with the soft detection technique is that the search spaces $S_{l,i}^-$ and $S_{l,i}^+$ expand exponentially with the number of layers and bits per symbol. Before we compute the bit-level LLR for the MIMO soft output, we will develop a new OSIC based algorithm to reduce the complexity of MIMO detection in the next section.

III. MODIFIED LAYERED OSIC ALGORITHM

In brief, the main concept of the conventional OSIC detection scheme is first to determine the optimal ordering for the layered signals, and then iteratively cancel the interference effects of other layered signals by the order from the highest post-detection signal to noise ratio (SNR) to the lowest one. The transmitted symbol is estimated from the received signal vector by removing the effects of other transmitted symbols with the order preceding the detecting symbol [5], [16]. The procedure of the OSIC algorithm can be described as follows.

At the initial detection stage, let i = 1 and then $\mathbf{H}(1) = \mathbf{H}$ and $\mathbf{y}(1) = \mathbf{y}$. At the *i*th detection stage, if considering MMSE filtering for channel estimation, a nulling matrix $\mathbf{G}(i)$ is calculated by

$$\mathbf{G}(i) = \left(\mathbf{H}(i)^{H}\mathbf{H}(i) + \frac{\sigma_{n}^{2}}{\sigma_{s}^{2}}\mathbf{I}\right)^{-1}\mathbf{H}(i)^{H},$$
(5)

where $(\cdot)^{H}$ indicates conjugate transpose, σ_{s}^{2} is the signal power for each transmit antenna and equals $1/N_{t}$ of the total transmission power. Considering matrix $\mathbf{H}(i)$ may be not square such as $m \times n$, the pseudoinverse computation will be used in (5) under the condition $m \ge n$. Instead of choosing an arbitrary ordering to detect, [16] suggested the optimal ordering with the minimum squared Euclidean norm of $[\mathbf{G}(i)]_{p}$:

$$\alpha_{i} = \arg\min_{p \notin \{\alpha_{1}, \alpha_{2}, \cdots, \alpha_{i-1}\}} \| [\mathbf{G}(i)]_{p} \|^{2}, \quad p = 1, 2, \cdots, N_{t}, \quad (6)$$

where $[\mathbf{G}(i)]_p$ denotes the *p*th row of matrix $\mathbf{G}(i)$. The vector $[\mathbf{G}(i)]_{\alpha_i}$ can then be used to null all but the optimally ordered

layer α_i signal. By using this nulling vector, the transmitted symbol of layer α_i can be detected within the constellation C by

$$\hat{\mathbf{g}}^{(\alpha_i)} = \operatorname*{arg\,min}_{s \in C} \|s - [\mathbf{G}(i)]_{\alpha_i} \mathbf{y}(i)\|^2. \tag{7}$$

Denote $\mathbf{Q}(\cdot)$ the QAM slicer for the signal constellation in use. Considering transmitted symbols are equally probable and the noise is independently Gaussian distributed, the detected symbol can be also obtained by

$$\hat{s}^{(\alpha_i)} = \mathbf{Q}\left([\mathbf{G}(i)]_{\alpha_i}\mathbf{y}(i)\right). \tag{8}$$

Once the layer *i* is detected, the interference resulting from $\hat{s}^{(\alpha_i)}$ can be canceled to improve detection of the subsequent layers by modifying the received vector $\mathbf{y}(i+1)$ at the next detection stage i+1 as

$$\mathbf{y}(i+1) = \mathbf{y}(i) - \hat{s}^{(\alpha_i)} \{ \mathbf{H}(i) \}_{\alpha_i}, \tag{9}$$

where $\{\mathbf{H}(i)\}_{\alpha_i}$ is the α_i th column of $\mathbf{H}(i)$. Since the layer α_i is detected, the channel matrix $\mathbf{H}(i+1)$ at the (i+1)th detection stage should be deflated by removing the α_i th column from $\mathbf{H}(i)$,

$$\mathbf{H}(i+1) = null < \mathbf{H}(i) >_{\alpha_i},\tag{10}$$

where $null < \cdot >_{\alpha_i}$ denotes the operation of nulling the α_i th column vector. The processes (5)–(10) repeat with i := i + 1 until all symbols are detected.

The conventional OSIC algorithm performs successive symbol cancellation to solve the layered signals step by step. Whenever the decision leads to an error, the OSIC detector will experience error propagation and its performance is significantly degraded compared to the result obtained by MLD. However, a tradeoff between detection performance and computational complexity is inevitable. Among the previously proposed methods, the SD approach has a good performance close to MLD. The SD algorithm gives a depth-first search on possible N_t -vector symbol candidates that lie within a hypersphere of a given radius around the received vector y. As one candidate is found, it is stored as the possible ML solution and a new search begins with a smaller radius which is updated with its associated Euclidean distance. The search process ends until no more candidates are found, and the last stored candidate is the ML solution. Although SD reduces the requirement of complexity somehow, its complexity is still considerable when the number of antennas is large along with high order of QAM. In fact, we found that not all of the layered symbols are required to be involved in the SD search if a little performance loss, say 0.5 dB, can be tolerated. The OSIC process can be applied to high SNR layers with a satisfying performance close to the ML.

To shrink the gap between OSIC and MLD, we propose a new method called modified layered OSIC algorithm which is composed of three parts. Since the number of layered symbols that are determined by searching are usually small, the new method simply considers the search paths based on a detection probability bounded region rather than the searching process with both forward and backward directions and search node enumeration techniques [29], [30] as employed in the SD algorithm. We will introduce it detailedly in the following content.





Fig. 2: Concept of the IL OSIC algorithm [21] with L=1 and (the proposed) modified layered OSIC algorithm with L=2.

Modified OSIC

Algorithm

Modified OSIC

Algorithm

Modified OSIC:

Nt-2 layers

A. Reliability Improvement of Weak Layers

Modified OSIC Algorithm

Modified OSIC Algorithm

Modified OSIC Algorithm

Modified OSIC Algorithm

As observed from the OSIC algorithm, we find that the performance is mostly limited by ill-conditioned layers. Instead of performing symbol cancellation with the optimum ordering for the best-first layered signal in each recursion of the OSIC algorithm, the work in [21] improved detection accuracy of the weakest layer by starting with the worst sub-channel and detecting the weakest layer using exhaustive search over the possible transmitted symbols. And then, the conventional OSIC scheme is applied to the remaining $N_t - 1$ layers. The concept is depicted in Fig. 2(a). The number of the expanded OSIC branches following the weakest layer p is that of possible transmitted symbols in signal constellation.

Actually, it may be not enough to consider only the worst sub-channel to decrease the performance gap between OSIC and MLD, especially for a MIMO system with a large number of antennas. Let L be the number of layers chosen for symbol search excluded from performing OSIC. Assume that layers p_1, p_2, \dots, p_L are the L layers selected for symbol search. Considering MMSE channel estimation, the L layers are

chosen by

$$\{p_1, p_2, \cdots, p_L\} = \arg_{k_1, k_2, \cdots, k_L} \{ \|[\mathbf{G}]_i\|^2 \ge \|[\mathbf{G}]_j\|^2 \}, \quad (11)$$

where $i \in \{k_1, k_2, \cdots, k_L\}, j \in \{k_{L+1}, k_{L+2}, \cdots, k_{N_t}\}, \{k_1, k_2, \cdots, k_{N_t}\}$ is a permutation of $\{1, 2, \cdots, N_t\}$, and $\mathbf{G} = \left(\mathbf{H}^H \mathbf{H} + \frac{\sigma_n^2}{\sigma_s^2} \mathbf{I}\right)^{-1} \mathbf{H}^H.$

For a MIMO system with a small number of antennas, e.g., smaller than 4×4 which is considered in [21], L = 1 may be enough because only four layers can be chosen for symbol search. However, as the number of antennas increases, the number of layers for symbol search should be increased as well. Although L is generally small, as will be shown from our analysis, L = 2 can result in a significant performance improvement in the case of 8×8 MIMO. When L equals the number of layers, i.e., N_t , the OSIC algorithm vanishes and the performance obtained by only searching symbols is equivalent to that of MLD.

However, the method to add the symbol search preceding OSIC increases the branches to perform OSIC for the remaining layers. Exponentially growing complexity may reduce the benefits in practice, especially when the number of possible transmitted symbols becomes large, e.g., 64-QAM. Hence decreasing the number of search branches turns out to be an important issue in applying a value of L greater than unity. Fig. 2(b) depicts that some search branches are purged for the example of L = 2, in which the overall complexity can be dramatically reduced. Note that in this figure, the new OSIC algorithm called "modified OSIC algorithm" and how to purge the branches will be introduced in the next subsections.

B. Search Branches Reduction Scheme

It is a high probability for an erroneously detected symbol to be demodulated into one of its neighboring constellation points. To reduce the computational burden, we can take into account the search branches with the constellation points located within a predetermined region, provided that the center of the region takes the decided output as reference. The branches are simply purged for the corresponding constellation points located outside the region.

According to the modulation type and the decided output, there are different regions to set up the active search branches. Consider 16-QAM, for instance, the regular 16 circles in a constellation diagram are the possible symbols as depicted in Fig. 3. Suppose the regular circle marked with a outer circle is the decided symbol. Based on the possible position of the decided symbol, there are three cases to construct the regions which centers are at their decided symbols for a given radius D. The output symbol decided at the four corner positions is Case 1, where the number of the nearest neighbors is two. Case 2 is at the eight outermost positions excluding the four corners, where the number of the nearest neighbors is three. Located at the four innermost positions is Case 3, where the number of the nearest neighbors is four. For the same case, regions constructed by the same D contain the same number of symbols. That is, D determines how to choose the most possible symbols to launch the branches. Also as depicted in Fig. 3, we show eight types of regions for each case, and Type



Fig. 3: The eight types of regions for reducing the number of search branches depending on the value of D.



Fig. 4: Determination of the region types related to *D* for Case 3.

9 is the region covering all constellation points for all cases. The number of covered symbols increases gradually from Type 1 to Type 9.

In Fig. 4, we redraw the 16-QAM constellation diagram to illustrate how to determine those types defined in Case 3, for instance. The Type 1 region contains minimum symbols which are closest to the decided symbol. As we set the distance between two neighboring symbols as 2, the value of D for

Type 1 is 2. Then, D for Type 2 is chosen as $2\sqrt{2}$ because the region constructed by this radius covers four symbols that are next closest to the Type 1 region. By this way, D is 4 to construct Type 3 in which two more symbols are included, and likewise, we can get the values of D for Type 4 and Type 5 as $2\sqrt{5}$ and $4\sqrt{2}$ to contain more four symbols and the most distant symbol, respectively. Notice that, in Case 1 and Case 2, more symbols are possibly located far away from the decided symbols such that D can have larger values and then we have nine types as listed in Table I. However, the result is the same from Type 5 to Type 9 for Case 3 since all of the 16 symbols has been covered by the radius defined in Type 5.

It is worthy of note that D is determined in terms of the Euclidean distance. One more thing deserving to note is that D determines the region type, i.e., how many symbols out of the whole symbol set can be discarded. Here, we briefly define a new parameter R instead of D, where $R = 1, 2, \dots, 9$ represents Type 1, Type 2, \dots , Type 9, respectively.

Let W be the number of the chosen branches in our scheme and w the branch index with $w = 1, 2, \dots, W$. Denote the chosen symbol for layer p_i to be improved by $\tilde{s}_w^{(p_i)}, i = 1, 2, \dots, L$. The new input to the OSIC algorithm becomes

$$\mathbf{y}_w = \mathbf{y} - \sum_{i=1}^{L} \tilde{s}_w^{(p_i)} \{ \mathbf{H} \}_{p_i}, \tag{12}$$

where $\{\mathbf{H}\}_{p_i}$ denotes the p_i th column of matrix **H**. The new

Region (<i>R</i>) Distance (<i>D</i>)	Type 1 2	Type 2 $2\sqrt{2}$	Type 3 4	Type 4 $2\sqrt{5}$	Type 5 $4\sqrt{2}$	Type 6 6	Type 7 $2\sqrt{10}$	Type 8 $2\sqrt{13}$	Type 9 $6\sqrt{2}$
Case 1	3	4	6	8	9	11	13	15	16
Case 2	4	6	8	11	12	13	15	16 16	16 16
Case 5	3	9	11	15	10	10	10	10	10
PSSB(%)	75.0	60.9	48.4	29.7	23.44	17.2	7.8	4.7	0

Table I: The values of W_{ij} and PSSB with 16-QAM modulation.

channel matrix requires to be deflated as

$$\mathbf{H} := null < \mathbf{H} >_{\{p_1, p_2, \cdots, p_L\}},\tag{13}$$

where $null < \cdots >_{\{p_1, p_2, \cdots, p_L\}}$ denotes the operation of removing column vectors at columns p_1, p_2, \cdots , and p_L . There are W possible input results \mathbf{y}_w required for evaluation in OSIC. However, W is less than the total number of performing exhaustive search, N_S^L , where N_S is the number of constellation points.

To quantitatively analyze the proposed scheme, we may define a new measure, *Percentage of Saved Search Branches* (PSSB), to indicate the efficiency of the proposed search branches reduction scheme for different types. In general, denote W_{ij} the number of the symbols contained in the region of Type #i for Case #j, P_j the probability of transmitting symbols for Case #j, N_C the number of cases from our definition. The PSSB can be calculated by

$$PSSB(\%) = \sum_{j=1}^{N_C} \frac{(N_S - W_{ij})P_j}{N_S} \times 100\%.$$
 (14)

Here, the parameters for 16-QAM are $P_1 = 1/4$, $P_2 = 1/2$, $P_3 = 1/4$, $N_C = 3$, and $N_S = 16$. PSSBs for the nine types are also listed in Table I. From this table, we can see that the PSSB of Type 9 is zero because Type 9 explores all search branches before performing OSIC. The result is equivalent to implementing the ML scheme, which is the optimal performance provided with the penalty of complexity. As we choose the region of small type number, the computational complexity can be saved at the cost of degraded performance. From our simulation results for 16-QAM in the Rayleigh fading channel, the performance of Type 4 is quite close to that of MLD along with an almost 30% saving of complexity for L = 1 and 51% saving for L = 2.

C. Modified OSIC Algorithm

In order to reach a new efficient OSIC algorithm, we modify the SIC algorithm to reduce the requirement of computing pseudoinverse. Revisiting the OSIC algorithm (5)–(10), suppose the *i*th recursion is being executed in OSIC and then we have $N_t - i + 1$ layers left for detection. Denote $SNR^{(1)}, SNR^{(2)}, \dots, SNR^{(N_t-i+1)}$ their postdetection SNRs in descending order. Assume that the postdetection SNRs of N_p layers out of the remaining $N_t - i + 1$ layers approach $SNR^{(1)}$ within a predefined threshold value μ (dB) and $\mu \geq 0$, i.e.,

$$SNR^{(1)} - SNR^{(j)} \le \mu, \quad j = 1, 2, \cdots, N_p + 1,$$
 (15)

where $0 \le N_p \le N_t - i$. As a smaller μ is chosen, the $N_p + 1$ layers have more similar post-detection SNR strength with the highest priority for interference cancellation. In fact, the optimal ordering rule in the OSIC algorithm is to choose the layer with the largest post-detection SNR, however, the performance of the OSIC algorithm is mostly limited by ill-conditioned layers. The new algorithm is based on that only little impact will be caused if we alter the ordering of the high priority layers with a similar SNR strength. Ignoring the concern of ordering in this condition, we can execute the *i*th recursion by simultaneously canceling interferences coming from the $N_p + 1$ layers. By this means, N_p pseudoinverse operations are saved.

In this modified OSIC method, provided that $N_p + 1$ layers are merged in the same recursion, we describe the new OSIC algorithm as follows.

Initialization. The initial stage at i = 1 is to set $\mathbf{H}(1) = \mathbf{H}$ and $\mathbf{y}(1) = \mathbf{y}$.

Optimal Ordering. For the *i*th detection stage, the MMSE nulling matrix and the optimal ordering are calculated by (5) and (6).

Detection Merging. Then we evaluate the post-detection SNRs for non-detected layer $p, p \notin \{\alpha_1, \alpha_2, \dots, \alpha_{i-1}\},\$

$$SNR_p(i) = \frac{\sigma_s^2}{\sigma_n^2 \|[\mathbf{G}(i)]_p\|^2}$$
(16)

and merge the detection layers if their SNRs satisfy

$$SNR^{(1)}(i) - SNR^{(j)}(i) \le \mu,$$
 (17)

where $j=1, 2, \dots, N_t - i + 1$, and μ is a pre-defined parameter with $\mu \ge 0$. Note that if only j = 1 satisfies the merging rule, it is the same with the conventional OSIC algorithm.

Parallel Slicing. Assume $N_p + 1$ layers can be merged. A "coarse" estimate of the $N_p + 1$ transmitted symbol vector can be calculated by

$$\begin{bmatrix} \tilde{s}^{(\alpha_i)}, \tilde{s}^{(\alpha_{i+1})}, \cdots, \tilde{s}^{(\alpha_{i+N_p})} \end{bmatrix}^T \\ = \mathbf{Q} \left(\left[[\mathbf{G}(i)]_{\alpha_i}^T, [\mathbf{G}(i)]_{\alpha_{i+1}}^T, \cdots, [\mathbf{G}(i)]_{\alpha_{i+N_p}}^T \right]^T \mathbf{y}(i) \right) \mathbf{18}$$

Note that the detected result in (18) is coarse because N_p similar-level interferences exist when solving any one of the $N_p + 1$ symbols even though i - 1 interferences have been subtracted through the former SIC recursions. To improve the result of this merged detection, we can cancel the mutual interferences for layer α_{i+j} , $j = 0, 1, \dots, N_p$, by

$$\tilde{\mathbf{y}}_{j}(i) = \mathbf{y}(i) - \sum_{\ell=0}^{j-1} \hat{s}^{(\alpha_{i+\ell})} \{\mathbf{H}(i)\}_{\alpha_{i+\ell}} - \sum_{\ell=j+1}^{N_p} \tilde{s}^{(\alpha_{i+\ell})} \{\mathbf{H}(i)\}_{\alpha_{i+\ell}}$$
(19)

and produce the detected symbol as

$$\hat{s}^{(\alpha_{i+j})} = \mathbf{Q}\left([\mathbf{G}(i)]_{\alpha_{i+j}}\tilde{\mathbf{y}}_j(i)\right),\tag{20}$$

where $\tilde{\mathbf{y}}_j(i)$ is a temporary variable denoting the modified received signal after removing mutual interferences for layer α_{i+j} at recursion *i*. Equation (19) can be viewed as a modified parallel interference cancellation method, in where the second term on the right hand side performs postcancellation because $\hat{s}^{(\alpha_{i+\ell})}$, $\ell = 0, 1, \dots, j - 1$ is the already detected signal while the third term performs precancellation because $\tilde{s}^{(\alpha_{i+\ell})}$, $\ell = j + 1, j + 2, \dots, N_p$ is the yet undetected signal.

Interference Cancellation. As the better estimates $\hat{s}^{(\alpha_i)}, \hat{s}^{(\alpha_{i+1})}, \cdots$, and $\hat{s}^{(\alpha_{i+N_p})}$ are obtained, the received signal for the subsequent recursion index $i+N_p+1$ is modified by

$$\mathbf{y}(i+N_p+1) = \mathbf{y}(i) - \sum_{\ell=0}^{N_p} \hat{s}^{(\alpha_{i+\ell})} \{\mathbf{H}(i)\}_{\alpha_{i+\ell}}.$$
 (21)

Channel Matrix Deflating. The new channel matrix $\mathbf{H}(i + N_p + 1)$ is updated by deflating $\mathbf{H}(i)$ with simultaneously removing column vectors at columns α_i , α_{i+1} , \cdots , α_{i+N_p} , i.e.,

$$\mathbf{H}(i+N_p+1) = null < \mathbf{H}(i) >_{\{\alpha_i,\alpha_{i+1},\cdots,\alpha_{i+N_p}\}} .$$
(22)

The processes (5), (6), and (16)–(22) repeat with $i := i + N_p + 1$ until all symbols are detected.

In the new algorithm, partial layers can be removed from the received signal in parallel if their post-detection SNRs are so close that they can be treated as the most possible candidates to perform interference cancellation in next recursions. If little performance loss due to partially simultaneous interference cancellation can be tolerated compared to the SIC method, the proposed algorithm will benefit from reducing complexity.

D. Summary and Soft Detection Algorithm

Now we summarize the modified layered OSIC algorithm as follows :

• STEP 1

By (5), we initialize the nulling matrix $\mathbf{G}(1)$ for N_t layers. Suppose we enhance L worst layers, so that from (11) layers p_1, p_2, \dots, p_L are picked for use in the search branches reduction scheme.

• STEP 2

Determine R for reducing the number of search branches and pick those constellation points $\{\tilde{s}_w^{(p_j)}\}_{w=1}^W, j = 1, 2, \cdots, L$, within the search region for the L worst layers.

STEP 3

Suppose the *w*th branch contains the symbols picked from the *L* worst layers, $\tilde{s}_w^{(p_1)}, \tilde{s}_w^{(p_2)}, \dots, \tilde{s}_w^{(p_L)}$. Then we remove the symbol interferences due to the *L* worst layers

from the received signal vector \mathbf{y} and the new signal vector becomes $\mathbf{y}_w = \mathbf{y} - \sum_{j=1}^L \tilde{s}_w^{(p_j)} \{\mathbf{H}\}_{p_j}$.

• STEP 4

Once the parameter μ is given, we execute the modified OSIC algorithm for the remaining $N_t - L$ layers based on the new signal vector \mathbf{y}_w . Then the modified OSIC algorithm produces the remaining symbol set denoted as $\{\tilde{s}_w^{(p_j)}\}_{w=1}^W$, where $j = L + 1, L + 2, \cdots, N_t$.

• STEP 5

As we obtain all of the candidate symbol vectors $\tilde{\mathbf{s}}_w = [\tilde{s}_w^{(1)} \ \tilde{s}_w^{(2)} \cdots \tilde{s}_w^{(N_t)}]^T$, $w = 1, 2, \cdots, W$, the best choice can be made by evaluating the following likelihood function over the W search branches:

$$\hat{\mathbf{s}} = \underset{w=1,2,\cdots,W}{\operatorname{arg\,min}} \|\mathbf{y} - \mathbf{H}\tilde{\mathbf{s}}_w\|^2.$$
(23)

As mentioned about coded V-BLAST in Section II, we know how to convert the output of detected symbols to softdecision through (4). But in an SM-MIMO system, the total number of elements in the candidate vector sets, $S_{l,i}^+$ and $S_{l,i}^-$, relies on the numbers of transmit and receive antennas, (N_t, N_r) , and the size of modulation dimension. As the number of antennas and the size of modulation dimension increases, the complexity of computing LLR increases as well. For instance, assume the numbers of antennas are $N_t = N_r =$ 4 and the modulation type is 16-QAM, then the number of elements in $|\mathbf{S}_{l,i}^+|$ and $|\mathbf{S}_{l,i}^-|$ becomes $16^4/2 = 32768$, which implies a very high computational complexity to calculate the bit-level LLR.

Here, we denote by V a set of candidate symbol vectors obtained by the modified layered OSIC algorithm. Then (4) can be simplified as

$$LLR(b_{l,i}|\mathbf{y}) \approx \frac{1}{2\sigma_n^2} \min_{\mathbf{s} \in \mathbf{S}_{l,i,\mathbf{v}}^-} \|\mathbf{y} - \mathbf{Hs}\|^2$$
$$-\frac{1}{2\sigma_n^2} \min_{\mathbf{s} \in \mathbf{S}_{l,i,\mathbf{v}}^+} \|\mathbf{y} - \mathbf{Hs}\|^2, \quad (24)$$

where $\mathbf{S}_{l,i,\mathbf{V}}^+$ and $\mathbf{S}_{l,i,\mathbf{V}}^-$ stand for the sets of candidate symbol vectors that are "1" or "0" for bit l on the *i*th transmit antenna, respectively. However, it may come to the case that the candidate vector set is possibly empty, i.e.,

$$\mathbf{S}_{l,i,\mathbf{V}}^{-} = \mathbf{S}_{l,i}^{-} \cap \mathbf{V} = \phi \quad \text{or} \quad \mathbf{S}_{l,i,\mathbf{V}}^{+} = \mathbf{S}_{l,i}^{+} \cap \mathbf{V} = \phi.$$
(25)

This will lead to a computing problem for either term in (24). However, not both $\mathbf{S}_{l,i,\mathbf{V}}^-$ and $\mathbf{S}_{l,i,\mathbf{V}}^+$ are simultaneously empty. Thus, in order to avoid the special case in (25), we can depute an arbitrary value Γ to the term with an empty set for the bitlevel LLR value. As we obtain the bit-level LLR output, a SISO channel decoder can be then applied.

IV. PERFORMANCE DISCUSSION

A. Average Symbol Error Probability of Search Branches Reduction

Here, we explore the influence of the search branches reduction method on performance. The tradeoff of complexity and performance relies on the type determined by R. Larger R reduces the symbol error rate (SER). Once the transmitted symbol is given, the SER performance due to the symbols

lying out of the search region can be evaluated. Consider the 16-QAM constellation for Type 4 in Fig. 3 with average symbol energy E_s and the possible one-dimensional symbol values are $\{\pm a, \pm 3a\}$, where $a = \sqrt{E_s/10}$. The decision error is assumed to be associated with the Gaussian noise with zero mean and a variance $N_0/2$ for simplicity. Since there are three cases, case I, II, and III, for each type, the symbol error probability in the search layer is

$$P_e = 1 - \left[\frac{1}{4}P(C|\mathbf{I}) + \frac{1}{2}P(C|\mathbf{II}) + \frac{1}{4}P(C|\mathbf{III})\right].$$
 (26)

Define the Gaussian Q-function as $Q(u) = \int_u^\infty \frac{e^{-y^2/2}}{\sqrt{2\pi}} dy$. The probabilities P(C|I), P(C|II), and P(C|III) are given by

$$P(C|\mathbf{I}) = \left[1 - Q\left(\sqrt{\frac{2 \cdot (5a)^2}{N_0}}\right)\right]^2$$
$$- \left[Q\left(\sqrt{\frac{2 \cdot (3a)^2}{N_0}}\right) - Q\left(\sqrt{\frac{2 \cdot (5a)^2}{N_0}}\right)\right]^2$$
$$= 1 - 2\beta - \alpha^2 + 2\alpha\beta \qquad (27)$$
$$P(C|\mathbf{II}) = 1 - Q\left(\sqrt{\frac{2 \cdot (3a)^2}{N_0}}\right) - Q\left(\sqrt{\frac{2 \cdot (3a)^2}{N_0}}\right)$$
$$\times \left[Q\left(\sqrt{\frac{2 \cdot (3a)^2}{N_0}}\right) - Q\left(\sqrt{\frac{2 \cdot (5a)^2}{N_0}}\right)\right]$$
$$= 1 - \beta - \alpha^2 + \alpha\beta \qquad (28)$$
$$P(C|\mathbf{III}) = 1 - \left[Q\left(\sqrt{\frac{2 \cdot (3a)^2}{N_0}}\right)\right]^2$$
$$= 1 - \alpha^2 \qquad (29)$$

where $\alpha = Q\left(\sqrt{\frac{1.8E_s}{N_0}}\right)$ and $\beta = Q\left(\sqrt{\frac{5E_s}{N_0}}\right)$. The symbol error probability can be approximated as

$$P_e = \beta + \alpha^2 - \alpha\beta \approx \mathcal{Q}\left(\sqrt{\frac{5E_s}{N_0}}\right). \tag{30}$$

Note that we choose L ill-conditioned layers for symbol search and the layered interference noises are also included in the overall noise variance in addition to the AWGN. Hence, E_s/N_0 in (30) is usually small in the search layer. Accordingly, the Type 4 region is a good choice from our numerical results. The average SER for the L search layers can also be evaluated. For example, let L = 2. Denoting by P_{e1} and P_{e2} the SERs of the first and the second search layers, respectively, the average SER for the two layers is

$$\bar{P}_e = \frac{1}{2} \left[2P_{e1}P_{e2} + (1 - P_{e1})P_{e2} + P_{e1}(1 - P_{e2}) \right].$$
(31)

B. Performance Influence of Weak Layers

Suppose L layers are picked in the search method, then $N_t - L$ layers are detected by the proposed OSIC algorithm. Let $s^{(1)}, s^{(2)}, \dots, s^{(N_t-L)}$ denote the symbols to be detected in sequence in the OSIC based algorithm. The $N_r \times 1$ signal vector for the kth OSIC layer is obtained by canceling the L search layers and the k - 1 preceding OSIC layers with

$$\mathbf{y}_k = \mathbf{y}_w - \sum_{i=1}^{k-1} \mathbf{h}_i \hat{s}^{(i)}, \qquad (32)$$

where $k = 1, 2, \dots, N_t - L$ and $\hat{s}^{(i)}$ denotes the previously detected symbols with $\mathbf{h}_i = {\mathbf{H}(i)}_{\alpha_i}$. For the undetected symbols, \mathbf{y}_k can be also written as

$$\mathbf{y}_k = \mathbf{h}_k s^{(k)} + \sum_{i=k+1}^{N_t - L} \mathbf{h}_i s^{(i)} + \mathbf{n}_k,$$
(33)

where the $N_r \times 1$ vector \mathbf{n}_k accounts for the kth OSIC layered noise and

$$\mathbf{n}_{k} = \sum_{i=1}^{L} \mathbf{h}_{p_{i}} \left(s^{(p_{i})} - \tilde{s}_{w}^{(p_{i})} \right) + \sum_{i=1}^{k-1} \mathbf{h}_{i} \left(s^{(i)} - \hat{s}^{(i)} \right) + \mathbf{v}$$

= $\mathbf{v}_{k} + \mathbf{v}$, (34)

where \mathbf{v}_k is the residual layered interference noise which is also known as the error propagation noise and \mathbf{v} is the complex AWGN. From (5), we denote by \mathbf{w}_k^H the nulling vector for detecting the *k*th OSIC layered symbol $s^{(k)}$ and $\mathbf{w}_k^H = [\mathbf{G}(k)]_{\alpha_k}$. To detect $s^{(k)}$, \mathbf{y}_k is multiplied by \mathbf{w}_k^H ; that is,

$$\mathbf{w}_{k}^{H}\mathbf{y}_{k} = \mathbf{w}_{k}^{H}\mathbf{h}_{k}s^{(k)} + \mathbf{w}_{k}^{H}\sum_{i=k+1}^{N_{t}-L}\mathbf{h}_{i}s^{(i)} + \mathbf{w}_{k}^{H}\mathbf{n}_{k}.$$
 (35)

As L layers are previously removed by the search method, the SNR for detecting the kth layered symbol becomes

$$\rho_{k}(L) = \frac{E||\mathbf{w}_{k}^{H}\mathbf{h}_{k}s^{(k)}||^{2}}{\sum_{i=k+1}^{N_{t}-L}E||\mathbf{w}_{k}^{H}\mathbf{h}_{i}s^{(i)}||^{2} + E||\mathbf{w}_{k}^{H}\mathbf{n}_{k}||^{2}}$$
$$= \frac{\mathbf{w}_{k}^{H}\mathbf{h}_{k}\mathbf{h}_{k}^{H}\mathbf{w}_{k}}{\mathbf{w}_{k}^{H}\left[\sum_{i=k+1}^{N_{t}-L}\mathbf{h}_{i}\mathbf{h}_{i}^{H} + \frac{\sigma_{n_{k}}^{2}}{\sigma_{s}^{2}}\mathbf{I}_{N_{r}}\right]\mathbf{w}_{k}}, \quad (36)$$

where $\sigma_{n_k}^2 = E[||\mathbf{v}_k||^2] + E[||\mathbf{v}||^2]$, \mathbf{I}_{N_r} is an $N_r \times N_r$ identity matrix. The optimum \mathbf{w}_k^H can be found based on maximizing $\rho_k(L)$, which actually becomes a generalized Rayleigh quotient problem and the maximum SNR $\rho_{k,\max}(L)$ is the maximum eigenvalue of $\left[\sum_{i=k+1}^{Nt-L} \mathbf{h}_i \mathbf{h}_i^H + \frac{\sigma_{n_k}^2}{\sigma_s^2} \mathbf{I}_{N_r}\right]^{-1} \mathbf{h}_k \mathbf{h}_k^H$.

Once the post-detection SNR is obtained, the SER for the kth layer, \mathcal{L}_k , can be expressed as

$$P_L(E|\mathcal{L}_k) = Q\left(\rho_{k,\max}(L)\right). \tag{37}$$

For simplicity, denote the SERs of the L detected symbols from the search branches reduction method by $P_L(E|\mathcal{L}_{Nt-L+1})$, $P_L(E|\mathcal{L}_{Nt-L+2})$, \cdots , $P_{\mathcal{L}}(E|\mathcal{L}_{Nt})$. The average SER of detecting symbols from the N_t layers can be evaluated as follows. Let τ represent the number of erroneously detected symbols in the N_t layers. The average SER with L search layers and $N_t - L$ OSIC layers can be calculated by

$$\bar{P}_L(E) = \frac{1}{N_t} \sum_{\tau=1}^{N_t} \tau \bar{P}_L(E|\tau),$$
(38)

where $\bar{P}_L(E|\tau)$ is the average SER given τ error symbols. Furthermore, to calculate $\bar{P}_L(E|\tau)$, we define by Ω_{τ} the space of collection of any different τ layers, which contains $C_{\tau}^{N_t}$ elements. Then,

$$\bar{P}_{L}(E|\tau) = \sum_{\Omega_{\tau}} P_{L} \left(E \left| \bigcap_{j \in \Omega_{\tau}} \mathcal{L}_{j} \right. \right) P_{L} \left(C \left| \bigcap_{\substack{j=1\\ j \notin \Omega_{\tau}}}^{N_{t}} \mathcal{L}_{j} \right. \right)$$
$$= \sum_{\Omega_{\tau}} \prod_{j \in \Omega_{\tau}} P_{L}(E|\mathcal{L}_{j}) \prod_{\substack{j=1\\ j \notin \Omega_{\tau}}}^{N_{t}} \left[1 - P_{L}(E|\mathcal{L}_{j}) \right],(39)$$

where we assume that the events in different layers are mutually independent and $P_L(C|\cdot)$ represents the correct probability with $P_L(C|\cdot) = 1 - P_L(E|\cdot)$. For example, when $\tau = 1, \Omega_1 = \{1, 2, \dots, N_t\}$ and

$$\bar{P}_{L}(E|\tau=1) = \sum_{i=1}^{N_{t}} \left(P_{L}(E|\mathcal{L}_{i}) \prod_{\substack{j=1\\j\neq i}}^{N_{t}} [1 - P_{L}(E|\mathcal{L}_{j})] \right)$$
$$= P_{L}(E|\mathcal{L}_{1}) \prod_{j=2}^{N_{t}} [1 - P_{L}(E|\mathcal{L}_{j})]$$
$$+ P(E|\mathcal{L}_{2}) \prod_{\substack{j=1,3,4,\cdots\\j=1,3,4,\cdots}}^{N_{t}} [1 - P_{L}(E|\mathcal{L}_{j})] + \cdots$$
$$+ P_{L}(E|\mathcal{L}_{N_{t}}) \prod_{j=1}^{N_{t}} [1 - P_{L}(E|\mathcal{L}_{j})]. \quad (40)$$

When $\tau = N_t, \, \Omega_1 = \{(1, 2, \cdots, N_t)\}$ and

$$\bar{P}_L(E|\tau = N_t) = \prod_{j=1}^{N_t} P_L(E|\mathcal{L}_j).$$
(41)

In the conventional OSIC algorithm, $P_L(E|\mathcal{L}_{Nt-L+1})$, $P_L(E|\mathcal{L}_{Nt-L+2})$, \cdots , $P_L(E|\mathcal{L}_{Nt})$ are obtained from the maximum eigenvalues of $\left[\sum_{i=k}^{Nt} \mathbf{h}_i \mathbf{h}_i^H + \frac{\sigma_{n_k}^2}{\sigma_s^2} \mathbf{I}_{N_r}\right]^{-1} \mathbf{h}_k \mathbf{h}_k^H$ for $k = N_t - L + 1$, $N_t - L + 2$, \cdots , N_t . The channel properties for ill-conditioned SNR layers drive the eigenvalues so small that $P_0(E|\mathcal{L}_k)$ is significantly larger than $P_L(E|\mathcal{L}_k)$ for $L \neq 0$, and thus, $\bar{P}_L(E) < \bar{P}_0(E)$. As we choose $L = N_t$, the proposed OSIC method becomes the ML solution. The choice of L compromises complexity and performance. From the viewpoint of setting the average SER within a promising bound, for instance, $\bar{P}_{e,\max}$, for a given SNR σ_s^2/σ_v^2 , a good choice of L can be

$$L = \arg \min_{\ell = 0, 1, \cdots, N_t} \{\ell : \bar{P}_{\ell}(E) < \bar{P}_{e, \max}\}.$$
 (42)

For example, L = 2 when $\bar{P}_{e,\max}$ is set as 0.5 dB SNR loss compared to that obtained from the ML for 8×8 QPSK with $\mu = 1$ and R = 4 as shown in Fig. 8(b).

C. BER with LLR

Consider an N_t layered MIMO detection problem, the received signal in (1) can be modeled by a linear combination

for each layered symbol as

$$\mathbf{y} = \sum_{j=1}^{N_t} \mathbf{h}_j s^{(j)} + \mathbf{n},\tag{43}$$

where $\mathbf{h}_j = {\{\mathbf{H}\}}_j$. For layer *i*, denote by \mathcal{I}_i the residual cancellation interference and by \mathbf{g}_i the corresponding MMSE weight with $\mathbf{g}_i = [\mathbf{G}]_i$. The output of the MMSE detector can be given as

$$\hat{s}^{(i)} = \mathbf{g}_i \mathbf{h}_i s^{(i)} + n'_i, \qquad (44)$$

where $n'_i = \mathbf{g}_i \mathcal{I}_i + \mathbf{g}_i \mathbf{n}$. For simplicity, assume the new noise n'_i is independent and complex Gaussian distributed with zero mean and variance $\sigma_{n'_i}^2$, where $\sigma_{n'_i}^2 = \|\mathbf{g}_i^T\|^2 (E_{I,i} + \sigma_n^2)$ and $E_{I,i}$ is the power of the residual interference. Let $\rho_i = \mathbf{g}_i \mathbf{h}_i$. For 16-QAM, the received signal prior to the soft detector becomes $\hat{s}^{(i)} = \rho_i s^{(i)} + n'_i$. The LLR for bit l, l = 1, 2, 3, and 4, can be simplified as

$$LLR(b_{l}^{(i)}|\hat{s}^{(i)}) = \frac{1}{\sigma_{\omega_{i}^{'}}^{2}} \left\{ \min_{s^{(i)} \in \mathbf{S}_{l,i}^{-}} \|\hat{s}^{(i)} - \rho_{i}s^{(i)}\|^{2} - \min_{s^{(i)} \in \mathbf{S}_{l,i}^{+}} \|\hat{s}^{(i)} - \rho_{i}s^{(i)}\|^{2} \right\}.$$
(45)

From [31], [32], the analytical expression for the average error probability of bit $b^{(i)}$ can be obtained as

$$P_{b^{(i)}} = \frac{3}{4} \overline{Q\left(\sqrt{\frac{4E_b|\rho_i|^2}{5N_0}}\right)} + \frac{1}{2} \overline{Q\left(\sqrt{\frac{36E_b|\rho_i|^2}{5N_0}}\right)} - \frac{1}{4} \overline{Q\left(\sqrt{\frac{100E_b|\rho_i|^2}{5N_0}}\right)}, \quad (46)$$

where $\overline{(\cdot)}$ denotes taking expectation on random variable ρ_i , E_b is the energy per transmitted bit, and $N_0/2$ is the power spectral density of noise n'_i . Note that in (46), the value ρ_i may simply approach unity as SNR becomes high and then the expectation operation can be omitted. However, $\sigma^2_{n'_i}$ is a function of \mathbf{g}_i and $E_{I,i}$. Since an exact analysis on $\sigma^2_{n'_i}$ is difficult, numerical results on LLR in comparison with ML will be performed in the next section.

V. COMPLEXITY AND NUMERICAL EVALUATION

In this section, the complexity and BER performance of the proposed algorithm are discussed and compared with the conventional OSIC algorithm and the (extended) IL OSIC algorithm. Modulation types including Gray-encoded QPSK and 16-QAM are considered in an 4×4 and 8×8 MIMO, in which the channels are mutually uncorrelated Rayleigh fading. The entries h_{ij} ($1 \le i \le N_r$ and $1 \le j \le N_t$) in the MIMO channel matrix **H** denote the channel gains modeled as independent complex Gaussian variables with unit variance, i.e., $E\{|h_{ij}|^2\} = 1$ [1].



Fig. 5: Comparison of pseudoinverse operation. (a) 4×4 16-QAM and L = 1. (b) 8×8 16-QAM and L = 2.

A. Complexity Analysis

The pseudoinverse operation in the OSIC algorithm significantly occupies the main computational complexity. As noted in [33], the pseudoinverse of an $N_t \times N_r$ matrix requires $3/2N_t^2N_r - 1/2N_tN_r$ multiplications and $3/2N_t^2N_r - 1/2N_t^2 - 3/2N_tN_r$ additions by using the Sherman-Morrison formula. First, we focus on the numbers of executing pseudoinverse for different cases to demonstrate the requirements of complexity. Since the merging process is not easy to predict, we evaluate the normalized number of employing pseudoinverse for each symbol through Mote Carlo simulation.

Fig. 5(a) shows the results with 4×4 MIMO 16-QAM at 20 dB SNR. The number for OSIC is 4 while that for IL OSIC is 49. In this case, L = 1 is enough for both 1L OSIC and the proposed algorithms. As μ increases, the number of employing pseudoinverse decreases for the proposed algorithm. Note that, when $\mu = 0$ dB, the modified OSIC algorithm uses conventional SIC and degenerates to the IL OSIC algorithm as R = 9. From BER results to be shown later, choosing R = 4 and $\mu = 4$ for the proposed algorithm. With these parameters, the proposed algorithm saves about 65% operation of matrix inversion compared to the IL OSIC algorithm.

Fig. 5(b) shows the results with 8×8 MIMO 16-QAM at 20 dB SNR. Now, the number of layers increases to 8. Hence the number of employing pseudoinverse becomes 8 for OSIC. In this case, our simulations showed that the original II OSIC algorithm does not give a satisfying performance anymore because of using L = 1. Then, the IL OSIC algorithm extends its search depth to L = 2 and so does the proposed algorithm. The number of employing pseudoinverse for the L = 2 extended IL OSIC algorithm is 1537. The proposed algorithm has a BER performance close to the L = 2 extended IL OSIC algorithm with R = 4 and $\mu = 1$. With these parameters, the proposed algorithm saves about 67% operation of matrix inversion compared to the L = 2 extended IL OSIC



Fig. 6: Comparison of FLOPs v.s. N_t for QPSK and 16-QAM modulation. We let R = 4 for 16-QAM, L = 1 and $\mu = 4$ for $N_t \le 4$ while L = 2 and $\mu = 1$ for $N_t \ge 4$.

algorithm.

The analysis of the average number of floating-point operations (FLOPs) is another insight into the comparison of complexity [26]. In Fig. 6, the numbers of FLOPs v.s. N_t are compared under the simulation performed with 16 dB SNR for QPSK and 20 dB SNR for 16-QAM. For conventional PIC and OSIC algorithms, the numbers of FLOPs have no difference for different modulation types. Since there is no search scheme to enhance the performance for ill-conditioned layers, PIC and OSIC have lower complexity and poorer performance compared to (Extended) IL OSIC and the proposed algorithms. For the proposed method, the numbers of FLOPs jump over those of the IL OSIC (L = 1) as $N_t \ge 5$ because L = 2is chosen to improve performance at the expense of a larger number of antennas, whereas the performance of the IL OSIC method degrades significantly when $N_t \ge 5$ in our simulation case. The L = 2 extended IL OSIC method has quite similar performance close to the proposed algorithm but its number of FLOPs is about 3.5 times more than that of the proposed method with 8×8 16-QAM.

B. Numerical Results

First, we explore the property of R in the search branches reduction scheme. Since the constellation points in QPSK are only four, reducing the number of search branches for QPSK is not valuable. We take into consideration 16-QAM in this



Fig. 7: Performance comparison for different R with L = 1. (a) 4×4 16-QAM. (b) 8×8 16-QAM.



Fig. 8: Performance comparison for different μ with R = 4. (a) 4×4 16-QAM and L = 1. (b) 8×8 QPSK and L = 2.

simulation. The result for 4×4 16-QAM is shown in Fig. 7(a). Here, we let $\mu = 0$ to evaluate the difference for different R. We can find that the BER performance of the proposed method approaches the MLD performance even when R = 4. Fig. 7(b) shows the result for a larger number of antennas, i.e., 8×8 16-QAM, where R = 4 results in a performance very close to that obtained by the IL OSIC method which is equivalent to our algorithm when R = 9 and $\mu = 0$. From the previous PSSB analysis, we may conclude that R = 4 is a good choice for 16-QAM with 29.7% saving of complexity for L = 1.

To explore the effect of μ , we set R = 4. Fig. 8(a) shows the results of the proposed algorithm for $\mu = 2, 4, 8$ (dB) in the case 4×4 16-QAM, where L = 1. In this case, the



Fig. 9: BER performance comparison with 4×4 and 8×8 QPSK modulation.

degradation due to the value of μ chosen in the modified OSIC algorithm is not significant even when a quite large value $\mu =$ 8. This implies that the performance of the conventional OSIC algorithm is greatly dominated by the weakest layer. As our algorithm uses the proposed search method for the weakest layer, only a little performance loss needs to be paid with $\mu = 8$. We next consider the effect with a larger number of antennas. Fig. 8(b) shows the results of the proposed algorithm for $\mu = 1, 3, 5$ (dB) in the case 8×8 QPSK with exhaustive search. Besides, we choose L = 2 instead of L = 1 in this case in order to achieve the performance close to MLD. We can see that the BER performance becomes a little affected by μ in the case of adding more antennas. Hence we will choose $\mu = 4$ (dB) for 4×4 antenna cases and $\mu = 1$ for 8×8 antenna cases in next simulation comparisons.

Fig. 9 shows the BER performance comparison of the proposed algorithm with IL OSIC, extended IL OSIC (L = 2), PIC, QRM-MLD [1], and ML methods for uncoded QPSK modulation over 4×4 antennas and 8×8 antennas. The PIC method has extremely low complexity but with very poor performance while the ML method has the best result but with unfeasible complexity. QRM-MLD can adjust its performance and complexity by choosing its decision-candidates parameter M. For M = 4 with QPSK, its performance and complexity are the same as ML. While M = 3, a worse performance than the modified OSIC algorithms presents in this simulation. Instead, IL OSIC (4×4) and extended IL OSIC (8×8) methods are more practical than ML and their performances



Fig. 10: BER performance comparison with 4×4 and 8×8 16-QAM modulation.

are quite close to those obtained by ML. The proposed algorithm has worse performance than the IL OSIC/extended IL OSIC methods only within 0.5 dB, but the requirement of pseudoinverse calculation can be saved up to 65% at 20 dB SNR. Fig. 10 shows the similar simulation results for 16-QAM modulation. It is worth to note that for the 8×8 case, the original IL OSIC performs worse than the proposed algorithm where we choose parameters L = 2, R = 4, and $\mu = 1$. As we extend the IL OSIC with L = 2, the extended IL OSIC method can achieve a satisfying performance. However, the proposed algorithm performs quite similar to the extended IL OSIC algorithm, but requires only 33% operation of matrix inversion.

The BER comparison for different N_t may be interesting for those OSIC based algorithms as shown in Fig. 11. It is apparent that search enhancement on only the weakest layer becomes a little insufficient as $N_t = 4$. The performance of the simple IL OSIC algorithm significantly deviates from those of the extended IL OSIC and the proposed algorithms as N_t is beyond 4. After $N_t \ge 5$, the proposed algorithm does not degrade as N_t increases because we choose L = 2based on (42). From the complexity issue shown in Fig. 6, the proposed algorithm does increase the FLOPS about double for QPSK and fourfold for 16-QAM as $N_t \ge 5$. Although the complexity of the proposed algorithm, compared to that of the simple IL OSIC, increases with a linear scale, the BER performance is improved significantly. Furthermore, $N_t = 5$ is the breaking point for the proposed algorithm beginning



Fig. 11: BER performance v.s. N_t with QPSK and 16-QAM modulation.

to use L = 2, where the BER performance seems not to be improved so much (as shown in Fig. 11). However, the performance of the proposed algorithm will not deviate since $N_t \ge 5$. The breaking point is actually a tradeoff relying on objective determination. In fact, this condition is due to L that we chose based on the setup of $\bar{P}_{e,\max}$ with 0.5 dB SNR loss from the ML result as described in (42).

In Fig. 12 we show BER performance comparison of the proposed algorithm and IL OSIC, both are tailored as soft-output detectors for coded QPSK/16-QAM modulation over 4×4 antennas. A binary rate 1/2 convolutional code with polynomials (133,171) in octal notation is used in our simulations. At the receiver, the soft detection output is then decoded by the Soft Output Viterbi Algorithm (SOVA) [34], [35]. From simulation results, we can see that the soft output algorithm for 4×4 QPSK and 16-QAM can gain about 4 to 6 dB at SNR.

VI. CONCLUSION

In this paper, we propose a modified layered OSIC detection algorithm for MIMO detection. The new algorithm brings down computational complexity through deflating the search tree of transmitted symbols in ill-conditioned layers to reduce the number of search branches and through introducing modified successive interference cancellation to reduce the number of calculating pseudoinverse. Compared with the MLD performance, the proposed algorithm is suboptimal for the purpose of feasibility. Although the L = 2 extended IL OSIC algorithm almost achieves the MLD performance in our simulation case, the proposed algorithm can employ a flexible L whereas the extended IL OSIC algorithm only applies a fixed L. Numerical results show that the new algorithm with proposed parameters can approach the performance obtained by the extended IL OSIC algorithm, along with saving a considerable amount of complexity.



Fig. 12: BER performance comparison with soft-detection for coded 4×4 QPSK and 16-QAM modulation.

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